TRAINING PACKAGE FOR TEACHING MATHEMATICS AT SECONDARY LEVEL

REPORT

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FOREWORD

Development, Review and finalisation of Training Package on Teaching Mathematics at Secondary Level is the outcome of two workshops held at Regional Institute of Education, Bhubaneswar from 18th to 22nd November 96 and from 10th to 14th March 1997. The State Level Coordination Committee of RIE/NCERT for Orissa had recommended the development of such a training package since it was felt necessary to orient secondary school teachers in the areas of Mathematics where the teachers have difficulty to understand and teach. The hard spots/difficult concepts in Mathematics at secondary level were identified through taking the views of 50 teachers drawn from various schools located in Orissa. It was found that the teachers had difficulty in understanding and comprehending a large number of concepts related to Real Numbers, Functions and Graphs, Surds, Loci and Concurrency theorems, Computing, Set theory - Venn diagram, Computing, Measures of Central Tendency, Arithmatic and Geometric progression etc. In subsequent workshop, the draft training material was reviewed by experts and teachers jointly. I appreciate the efforts of Dr.P.S.Tripathy, the Programme Coordinator and the faculty members of the Mathematics Department of this Institute in completing the task within the prescribed time limit. I hope that the package would help the teachers in transacting Mathematics concepts meaningfully.

28.01.1998

(Prof.D.K.Bhattacharjee)
PRINCIPAL

PREFACE

Training Package for Teaching Mathematics at Secondary level has been developed as a part of Programme Advisory Committee (PAC) approved programme. The purpose is to improve the quality of teaching - learning of Mathematics at Secondary level. This Training Package is intended for the use of Curriculum developers, teacher educators and other Key resource personnel working for the improvement of teaching - learning mathematics at Secondary level.

The views expressed by a large number of secondary school mathematics teachers in various inservice training programmes and the continued poor performance of the students in mathematics subject in the public examination conducted by Board of Secondary Education, Orissa prompted the Programme Coordinator to identify the difficult areas in secondary school mathematics for the teacher to teach and students to understand.

The HARD SPOTS in the text-books of mathematics at secondary level have been first identified taking the views of 45 teachers drawn from various schools in the State of Orissa through teacher's questionnaires. The analysis of responses to the teachers questionnaires revealed the following:

i) Teachers had difficulty in understanding and comprehending 4 concepts - three in Class IX and one in Class X.

ii) Teachers had difficulty in teaching 43 concepts - 36 in Class IX and 7 in Class X.

iii) As many as 44 concepts - 28 from Class IX and 16 from Class X were the difficult ones for teachers to understand and teach.

On synchronising the concept with the units in the text, it has been found that all the concepts identified as difficult ones by the practising teachers figure in mainly eight units of Class IX and X text-books of mathematics. The units are :

Class - IX

1. Real Numbers, Functions and Graphs.
2. Surds
3. Loci and Concurrency theorems
4. Computing
5. Set theory - Venn diagram

Class - X

1. Computing
2. Measures of Central tendency
3. Arithmetic and Geometric Progression.

In a workshop held at R.I.E., Bhubaneswar from November 18 to 22, 1996, nine practicising teachers, six subject experts, four external resource persons and the faculty members from Mathematics section of RIE, Bhubaneswar developed a draft training material on the identified hard spots in the areas of Algebra, Geometry, Statistics, Computing etc.

In a subsequent workshop held at RIE, Bhubaneswar between March 10 and 14, 1997, the draft training material was reviewed and revised by a team of Nine subject experts, three external resource persons and the faculty members from Mathematics section of RIE, Bhubaneswar.

Grateful thanks are due to Prof.D.K.Bhattacharjee, Principal, Prof.A.L.N.Sarma, Head, DESM, Dr.J.K.Mohapatra, Head, Department of Extension Education, Dr.K.K.Chakravarti, Dr.D.C.Sahoo, and to all others who made helpful suggestions and extended cooperation at various stages in the development of the training package.

I am very grateful to a large number of teachers who have responded to my request in a number of ways. They have given me the benefit of their experience and expertise . My thanks are also due to the participants of the training material development workshop who have given unstintingly of their time.

Finally my thanks are due to NCERT for assigning me, as a Coordinator of this PAC Programme, such a fascinating and enjoyable task.

P. S. Tripathi

P.S. Tripathi
Programme Co-ordinator

C O N T E N T S

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COMPUTING

APPENDICES

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ALJEBRA

S E T

1.1 Introduction: In our daily life we come across the expressions like "Tea Set", "Sofa Set" etc. But the word "Set" is now a days very widely used in mathematics. German mathematician Gerge Cantor had introduced the idea of Set in mathematics and since then the theory of Set has been playing an important role in making mathematics - simple, lucid and compact. Set theory has become the most important tool in teaching with mathematics at present.

1.2 Set and its elements:

Some mathematicians tried to define set as the collection of well defined objects. But some confusion arose on the word "Well defined" and ultimately it was decided to treat Set as self explanatory and accept Set as an undefined term.

When we say a "Tea Set" naturally we vidualise a collection of objects like a cup, a plate, a flask etc. These objects constitute a set and a cup or a plate or a flask is called an element or a member of the Set.

Normally we denote sets by capital letters A, B, C, D, S, X,Y, Z etc. and the objects or elements or members of a set by Lower-case letters a, b, c, d etc. We use the symbol " $\in$ " read as "an element of" or "belongs to" and the notation $\notin$ means "is not an element of" or "doesn't belong to". For example if W is the set of all

1.3 **How to Write a Set:** Usually one of the following two ways is used to describe a set:

a) Roster form or Tabular form

b) Set Builder form or Rule form.

a) **Roster form:-** In this form all the elements of the set are listed being separated by commas and are enclosed within a pair of braces. For example, the set of the days of a week is described in roster form as:

W = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

Similarly a set of natural members less than 6 is described as 5 = {1,2,3,4,5}. It is to be borne in mind that neither the repetition of elements nor the order of writing the elements affect the nature of the Set in any way. Thus {1,2,3,4,5,6} or {1,2,3,3,4,4,5,6, 6,6} or {5,2,1,4,3,6} is one and the same set.

b) **Set Builder form:-**

This form is used when all the elements of a set possess a common property which is not possessed by any element outside the said set. For example, in the set {a,e,i,o,u} all the elements possess a common property namely each of them is a vowel in English alphabet and no other letter possesses this property. Denoting this set in Set - builder form. We can write V = {x : x is a vowel in English alphabet} .

It is to be observed that we describe the Set by using a symbol x (any other symbol like y, z etc. could be used) which is followed by a colon " : " . After the sign of colon we write the characteristic property possessed by the elements of the set and then enclose the whole description within a pair of braces. The above description of the Set V is read as " V " is the Set of all x such that x is a vowel of English alphabet". In this description the colon stands for "such that " and the pair of braces stand for " the set of all".

For example, the following description of the Set $A \{ x : x$ is a natural number $2 < x < 7 \}$ is read as "the Set of all x such that x is a natural number and $2 < x < 7$. Hence the numbers 3, 4, 5, 6 are the elements of A. Thus $A = \{ 3, 4, 5, 6 \}$.

Both the above ways of description of Sets are useful and have their advantages. The following points are noteworthy.

a) Let us consider the $A = \{ 5,6,7,8,9, 10 \}$ (Roster form)

In set builder form A can be written as $A = \{ x : x \in N$ and $4 < x < 11 \}$ (Set builder form).

Set A is described in both methods.

b) (i) Let $B = \{$ Ram, Mahanadi, Pen, $\sqrt{5} \}$ (Roster form).

Here the roster form is preferable as it is difficult to find out a suitable common property of the elements of B.

c) Let us consider the Set of all sand particles on the seashore at Puri. Here the set builder form is preferable and we can write $S = \{x : x$ is a sand particle on the seashore at Puri$\}$.

1.4 Equality of Sets:-

Sets A and B are said to be equal (mentioned as $A = B$) if $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$ or in other words <u>two sets are said to be equal if they have the same elements</u>.

Example.1: $A = \{x : x^2 = 1\}$ and $B = \{1, -1\}$ are equal as $x^2 = 1 \Rightarrow x = \pm 1$. Hence $A = \{1, -1\}$

Example.2: $A = \{1,2,3,4\}$; $B = \{3,1,4,2\}$,
Here $A = B$ as both A and B have the same elements.

1.5 Finite and infinite Sets:-

If there is an end in counting the elements of a set, it is said to be a finite set and if there is no end in counting or if it is impossible to count the elements, then the set is said to be an infinite set.

Example.3: $A = \{1,2,3,4,5\}$
$B = \{x : x$ is an English alphabet$\}$
$C = \{3, 6, 9, 12, 15, 18, 21\}$

Here all the sets A, B and C are finite sets as A, B, C have respectively 5, 26 and 7 elements.

Example. 4: $N = \{1, 2, 3, 4, \ldots\ldots\}$

$Q = \{x : x = \frac{p}{q}, p, q \in N, q \neq 0\}$

$X = \{x : x \in R, 1 \leq x < 2\}$

All the sets N, Q and X are infinite sets .
Let us observe that in Set N we can count the elements but there is no end to it, and the elements of Q and X can't be counted.

1.6 Cardinality of a Set:-

Cardinality of a finite Set A is denoted by $|A|$, or n (A) or O(A) and is defined as the number of elements in A.

Example. 5: $A = \{a, b, c\}$

$\therefore \quad |A| = 3$

Example. 6: $B = \{p, q, q, p, r\}$

Here $|B| = 3$

1.7 Empty Set:- An empty set (or Null Set or Void Set) is defined as a set having no element. It is denoted by ϕ (read as phi).

Example. 7: $\phi = \{x : x \neq x\}$

Sometimes we use the symbol $\{\}$ to denote an empty Set

$\phi = \{$A week having 5 days$\}$

Note that $|\phi| = 0$

1.8 Subset and Super set:-

Let A and B be given Sets. If " $x \in A \Rightarrow x \in B$" it is said that A is a subset of B or B is a superset of A, and is denoted by $A \subseteq B$ or $B \supseteq A$ respectively. If A is not a subset of B, equivalently B is not a superset of A and is denoted by $A \nsubseteq B$ and $B \nsupseteq A$.

Note that (i) If $A \subseteq B$ and $B \subseteq A$, then $A = B$;

(ii) $A \subseteq A$ and (iii) ϕ is a subset of every set, i.e $\phi \subseteq A$.

Example. 8: $A = \{2, 4, 6\}$

$B = \{1, 2, 3, 4, 5, 6\}$

Here $A \subseteq B$

Example. 9: $A = \{4, 5, 6\}$

$B = \{1, 2, 3, 4, 5\}$

Here $A \not\subseteq B$

1.9 Universal Set:- If every Set in our discussion is a subset of a finite Set E, then E is said to be the universal set.

Let A_1 = {All people of Calcutta}

A_2 = {All people of Mumbai}

A_3 = {All people of Bhubaneswar}

E = {All people of India}.

Naturally each of A_1, A_2 and A_3 is a subset of E. Hence E is said to be the universal set of each of A_1, A_2 and A_3. One may choose E (universal set) as the people of Asia. This shows the universal set is not unique.

1.10 Power Set:- Set of all subsets of A is known as the Power Set of A and is denoted by P(A), So

$P(A) = \{x : x \subseteq A\}$

Note that every element of the power set of A is a subset of A.

Example. 10: Let $B = \{a, b, c\}$. All the possible subsets of B are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$ and B itself.

Hence $P(B) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b, c\}, \{a, b, c\}\}$

$P(\phi) = \phi$, [or $\{\ \}$] as phi is the subset of every set.

1.11 Cardinality of P(A) :-

$A = \phi \Rightarrow P(A) = \{\phi\} \Rightarrow |P(A)| = 1 = 2^0$

$A = \{a\} = P(A) = \{\phi, \{a\}\} \Rightarrow |P(A)| = 2 = 2^1$

$A = \{a,b\} \Rightarrow P(A) = \{\phi, \{a\}, \{b\}, \{a,b\}\} \Rightarrow |P(A)| = 4 = 2^2$

$A = \{a,b,c\} \Rightarrow P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b, c\}, \{a,b,c\}\} \Rightarrow |P(A)| = 8 = 2^3$

Now it is clear that

$|A| = 0 \Rightarrow |P(A)| = 2^0$

$|A| = 1 \Rightarrow |P(A)| = 2^1$

$|A| = 2 \Rightarrow |P(A)| = 2^2$

$|A| = 3 \Rightarrow |P(A)| = 2^3$

Hence we conclude that $|A| = n \Rightarrow |P(A)| = 2^n$ (where $n \in N$).

1.12 Equivalent Sets (Similar Sets):-

Two sets A and B are said to be equivalent (or similar) if we can associate exactly one element of B with every element of A and conversely can associate exactly one element of A with every element of B. In other words the sets A and B are equivalent if there is a one - to - one correspondence between the elements of A and B. If A and B are two

Note the following points:-

(i) $A \sim A$, Hence two equal sets A and B are equivalent but is the converse true ? i.e., if $A \sim B$, then we have $A = B$. It is not necessarily so. For example $A = \{a, b\}$, and $B = \{1, 2\}$. Here $A \sim B$ but $A \neq B$.

(ii) Two finite sets having the same cardinality are equivalent.

(iii) The set of natural numbers N and the set S of all even + Ve integers are equivalent.

For $N = \{1, 2, 3, 4, \ldots\}$

$S = \{2, 4, 6, 8 \ldots\}$ and we have the one - to - one association.

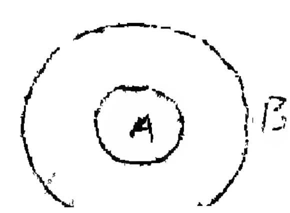
$1 \leftrightarrow 2,\ 2 \leftrightarrow 4,\ 3 \leftrightarrow 6,\ 4 \leftrightarrow 8$ and so on.

1.13 Venn - Diagram:-

In order to understand and vidualise the idea and concepts like Subset, Superset etc. we take the help of certain geometrical diagram. Such a diagram is known as Venn - Diagram after the name of John - Venn (1839 - 1883). When we show the Set A in a diagram, we mean that the elements of A are in the interrior of the closed curve but not on its boundary.

Case 1:- All the elements of A are in B.

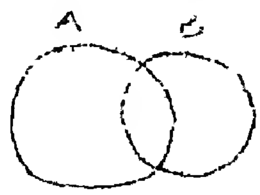
Hence $A \subset B$



Case ii :-

No element of A is in B nor any element of B is in A, i.e. $A \not\subset B$ and $B \not\subset A$.

Case iii :-



Every element of A is not in B and every element of B is not in A. Hence $A \not\subset B$ or $B \not\subset A$.

1.14 Set Operation:-

If x and y are two real numbers, then $x + y$, $x - y$, xy are also real numbers. Here each of the operations is known as binary operation.

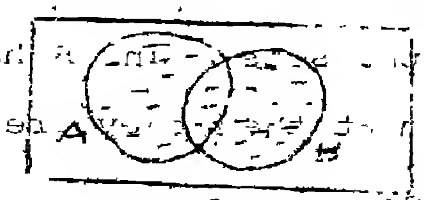
As +, - , X are binary operations in real numbers, similarly union, intersection, difference are the binary operations over the sets. The Algebra developed on sets taking these operations into account is known as Bulloon Algebra after the name of the famous mathematician George Boole (1815 - 1866).

Union:-

Let A and B be any two sets. Then A union B is defined as the collection of all the elements of A as well as of B. It is symbolically denoted by $A \cup B$.

Hence $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

This statement means x belongs to A or B or both A and B. $A \cup B$ can be illustrated through Venn - Diagram as follows:



Example. 11: Let $A = \{a, b, c\}$, $B = \{p, q, r\}$

$A \cup B = \{a, b, c, p, q, r\}$

Example. 12: $A = \{a, b, c\}$, $B = \{b, c, d, e\}$

$A \cup B = \{a, b, c, d, e\}$

<u>Some Important ideas on Union:</u>

(i) $A \subset B \Rightarrow A \cup B = B$

(ii) $A \cup A = A$

(iii) $A \cup \phi = A$

(iv) $A \subset (A \cup B)$ so also $B \subset A \cup B$.

It is clear from the definition of subset.

(v) $A \cup B = B \cup A$

i.e. Union operation is commutative.

It is clear from the definition itself.

Example. 13: $A = \{1, 2\}$, $B = \{p, q\}$

$A \cup B = \{1, 2, p, q\}$, $B \cup A = \{p, q, 1, 2\}$

$\Rightarrow A \cup B = B \cup A$

(vi) If A, B, C are three sets

Then $(A \cup B) \cup C = A \cup (B \cup C)$

That is Union operation is associative.

Example. 14: Let $A = \{a, b, c\}$, $B = \{b, c, d\}$,

$C = \{c, d, e\}$

Now $(A \cup B) \cup C = (\{a, b, c\} \cup \{b, c, d\})$

$\cup \{c, d, e\}$

$= \{a, b, c, d\} \cup \{c, d, e\} = \{a, b, c, d, e\}$

Again $A \cup (B \cup C)$

$= \{a,b,c\} \cup (\{b,c,d\} \cup \{c,d,e\})$

$= \{a,b,c\} \cup \{b,c,d,e\}$

$= \{a,b,c,d,e\}$

Hence $(A \cup B) \cup C = A \cup (B \cup C)$

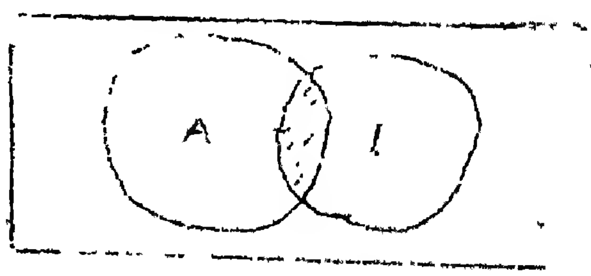
Intersection:

Let A and B be any two sets. Then A intersection B is defined as the collection of all the elements which are common to both.

It is symbolically denoted by $A \cap B$.

Hence, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$A \cap B$ can be illustrated through Venn - Diagram as follows



Example. 15:

Let $A = \{a, b, c, d\}$, $B = \{c, d, e\}$

$\Rightarrow A \cap B = \{c, d\}$

If the sets A and B have no common element then they are said to be disjoint sets.

or, $A \cap B = \phi$.

Example. 16:

$A = \{1, 2, 3\}$, $B = \{7, 11, 15\}$

$A \cap B = \{1, 2, 3\} \cap \{7, 11, 15\} = \phi$.

Some important ideas on Intersection of Sets:

(i) $A \subseteq B \Rightarrow A \cap B = A$

(ii) $A \cap A = A$

(iii) $A \cap \phi = \phi$

(iv) $(A \cap B) \subseteq A$ so also $(A \cap B) \subseteq B$

It is evident from the definition of intersection.

(v) $A \cap B = B \cap A$

i.e. Intersection operation is commutative. It is clear from the definition.

(vi) If A, B, C are three sets

Then $(A \cap B) \cap C = A \cap (B \cap C)$

i.e. Intersection operation is associative.

Let $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{c, d, e\}$

$(A \cap B) \cap C = (\{a,b,c\} \cap \{b,c,d\}) \cap \{c,d,e\}$

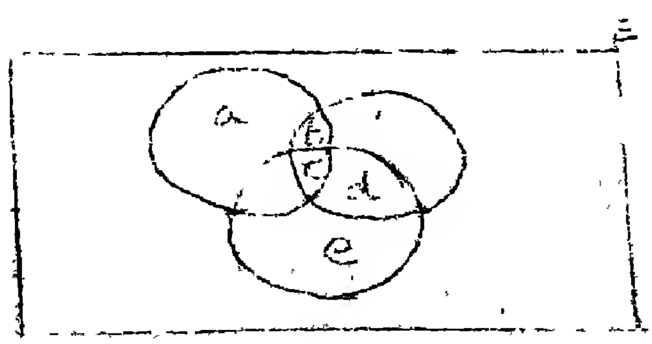
$= \{b,c\} \cap \{c, d, e\} = \{c\}$

$A \cap (B \cap C) = \{a,b,c\} \cap (\{b,c,d\} \cap \{c,d,e\})$

$= \{a, b, c\} \cap \{c, d\} = \{c\}$

Hence $(A \cap B) \cap C = A \cap (B \cap C)$

This can be shown through the Venn Diagram as follows:



(iii) <u>Difference of Sets:-</u>

Let A and B be two sets. Then the elements which are only in A but not in B. Constitute the Set A - B and is read as A difference B.

Hence $A - B = \{x \mid x \in A \text{ and } x \notin B\}$

Similarly $B - A = \{x \mid x \in B \text{ and } x \notin A\}$

Some illustrations of A-B taking different situations of A and B.

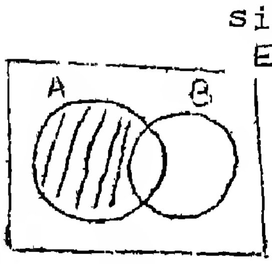


Fig I

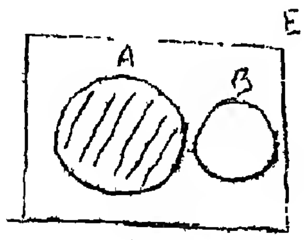


Fig. II

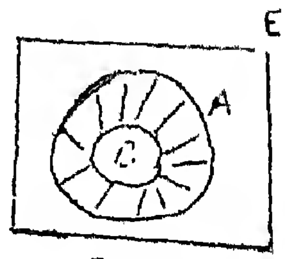


Fig. III

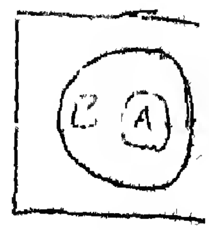


Fig. IV

A - B is the shaded portion:

In Fig. I, $A \cap B \neq \phi$ and $A \not\subset B$, $B \not\subset A$

in Fig.II, $A \cap B = \phi$ and $A \not\subset B$, $B \not\subset A$

In Fig.III. $B \subset A$

In Fig. IV. $A \subset B$

<u>Some Important ideas on difference of Sets.</u>

(i) $A - A = \phi$

It is evident from the definition .

(ii) $A - B \neq B - A$

Example. 17:

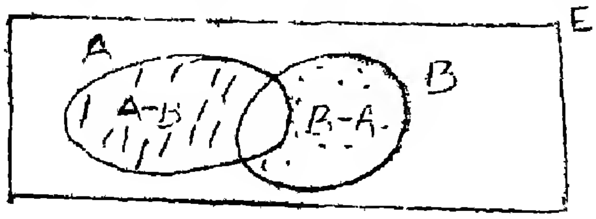
Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$

$A - B = \{1, 2\}$ and $B - A = \{5, 6\}$

Hence, $A - B \neq B - A$

i.e. The difference operation is not

through Venn diagram as follows:



It is clear from the Venn diagram that

(a) $(A-B) \cap (B-A) = \phi$,

(b) $(A-B) \cap (A \cap B) = \phi$,

(c) $(B-A) \cap (A \cap B) = \phi$,

(d) $A \cup B = (A-B) \cup (A \cap B) \cap (B-A)$.

(e) $A - B = A - (A \cap B)$

(f) $B - A = B - (A \cap B)$

iii) If A, B, C are three Sets then

$(A - B) - C \neq A - (B - C)$, that is

difference operation is not associative.

Example: 18:

Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5, 6\}$,

$C = \{3, 5, 6\}$

$A - B = \{1, 2, 3\} - \{3, 4, 5, 6\} = \{1, 2\}$

$B - C = \{3, 4, 5, 6\} - \{3, 5, 6\} = \{4\}$.

$(A-B) - C = \{1, 2\} - \{3, 5, 6\} = \{1, 2\}$

$A - (B-C) = \{1, 2, 3\} - \{4\} = \{1, 2, 3\}$

Hence, $(A-B) - C \neq A - (B - C)$

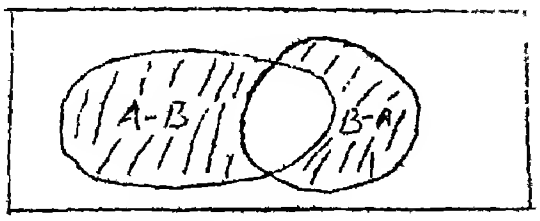
iv) <u>Symmetric Difference</u>:-

If A and B are two sets, then the symmetric between A and B is defined as the union of the sets $(A-B)$ and $(B-A)$. This denoted by $A \triangle B$.

i.e. $A \triangle B = (A-B) \cup (B - A)$

$A \triangle B$ is read as ' A delta B'

$A \triangle B$ is illustrated through Venn-diagram as follows:



Another definition of $A \triangle B$ is given as the set of elements of $(A \cup B)$ which are not in $(A \cap B)$

i.e. $A \triangle B = (A \cup B) - (A \cap B)$

Example. 19:

Let $A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$

$A - B = \{1,2\}$, $B - A = \{5, 6\}$,

$A \triangle B = (A-B) \cup (B-A) = \{1,2\} \cup \{5, 6\} = \{1,2,5,6\}$

Again $A \cup B = \{1,2,3,4\} \cup \{3,4,5,6\} = \{1,2,3,4,5,$

$A \cap B = \{1,2,3,4\} \cap \{3,4,5,6\} = \{3,4\}$

$A \triangle B = (A \cup B) - (A \cap B) = \{1,2,3,4,5,6\} - \{3,4\}$

$= \{1, 2, 5, 6\}$

Thus we see

$(A-B) \cup (B-A) = (A \cup B) - (B-A)$ are equivalent.

Some important ideas on symmetric difference

$A \triangle B = B \triangle A.$

Proof, $A \triangle B = (A-B) \cup (B-A) = (B-A) \cup (A-B)$

$= B \triangle A.$

i.e. Symmetric difference is commutative

v) <u>Compliment of a Set:</u>

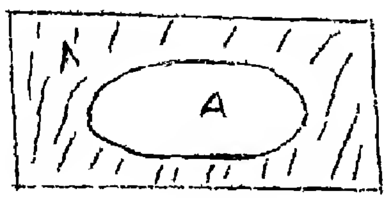
Let E be an universal set and A be one of its subset. Then complement of set A is defined as the set of elements belonging to E but not to A.

It is denoted by A'.

Hence $A' = E - A = \{x \mid x \in E \text{ and } x \notin A\}$

A' is read as ' A dash' .

Representation of A' through Vena diagram is as follows.



Example.20:

$E = \{1,2,3,4,5,6,\ 7\}$, $A = \{1,2,3\}$

$A' = E - A = \{1,2,3,4,5,6,7\} - \{1,2,3\}$

$= \{4,5,6,7\}$

Some important ideas on complementation

i) A and A' are always disjoint

i.e. $A \cap A' = \phi$

ii) $A \cup A' = E$

iii) $A' = E - A$ and $A = E - A'$

iv) $E' = \phi$ and $\phi' = E$

v) $(A')' = A$

vi) $|E| = m$ and $|A| = n \Rightarrow |A'| = m - n$

One of the important application of complementation of set is De Morgans Law.

De Morgans Law

i) $(A \cup B)' = A' \cap B'$

i.e. complement of the union of the sets is equal to the intersection of their complementation.

ii) $(A \cap B)' = A' \cup B'$

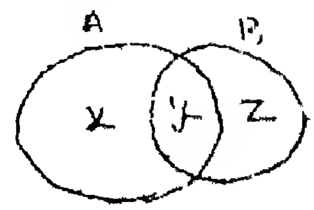
i.e. Complementation of the intersection of sets is equal to the union of their complementation.

The law is also true for the finite, or countable infinite sets.

Cardinality or order of the union of two sets A and B

$|A \cup B| = |A| + |B| - |A \cap B|$

Proof:



Let $|A| = x + y$

$|B| = y + z$

and $|A \cap B| = y$

It is clearly mentioned in the figure.

From the figure

$$|A \cup B| = x + y + z = (x+y) + (y + z) - y$$

$$= |A| + |B| - |A \cap B|$$

$$A \cap B = \phi \Rightarrow |A \cup B| = |A| + |B|$$

Cardinality of union of three sets A, B and C

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C|$$

$$|C \cap A| + |A \cap B \cap C|$$

Cartesian Product of two Sets:

If A and B are two non-empty sets then Cartesian product of A and B is defined as the set of elements of the ordered pair of the form (x, y)

where $x \in A$ and $y \in B$. Symbolically it is denoted by $A \times B$.

Hence $A \times B = \{ (x, y) \mid x \in A \text{ and } y \in B \}$

Example.21:

Let $A = \{1, 2, 3\}$

$B = \{5, 6\}$

Then $A \times B = \{1, 2, 3\} \times \{5,6\}$

$= \{(1,5), (1,6), (2,5), (2,6), (3,5), (3,6)\}$

$B \times A = \{5,6\} \times \{1, 2, 3\}$

$= \{(5, 1), (5, 2), (5,3), (6,1), (6,2) (6,3)\}$

Thus we see

$A \times B \neq B \times A$

A^2 is defined as $A \times A$

Example. 22:

Let $A = \{1, 2, 3\}$

$A^2 = \{1,2,3\} \times \{1, 2, 3\}$

$= \{(1,1), (1,2), (1,3), (2,1), (2,2),$

$(2,3), (3,1), (3,2), (3,3)\}$

$|A| = m, \ |B| = n \Rightarrow |A \times B| = |A| \times |B| = mn$

REAL NUMBER SYSTEM

2.1 Gradual development:

Emergence of natural numbers:

The first number which the man must have thought of using is the number 1. The first concept of quantifying thing must have been through 'One' and 'many'. Subsequently, the inexact, ness of quantification by the concept 'many' must have been improved by emergence of the counting numbers 1, 2, 3 and so on. However, the study of the 'number system and their properties is of prime importance for the human beings of the present days. The statement, "Mathematics is the queen of all subjects, but the study of the number system is the queen of Mathematics," made by Karl Fred ric Gouse speaks of its importance. The German Mathematician Leo Pol Cronechar said, 'None can say what God had created. If any one says that God created man, then he also created numbers along with man". This shows how intimately the numbers are associated with man.

Thus the Natural numbers were first created. Some important axioms related to the Natural number set (N) are as follow.

i) 1 is the smallest element of N

ii) $M \in N \Rightarrow m^+ \in N$

where m^+ is the successor of m^+

Thus the elements of N are -

$1, 1^+, (1^+)^+, \{(1^+)^+\}^+$, and so on.

using the symbols 2 for 1^+, 3 for $(1^+)^+$ etc. get, $N = \{1, 2, 3, \ldots\ldots\ldots\}$

Further because of (ii) we come to know that **there** is no last element in N.

Addition in N

(iii) $m^+ = m + 1$

(iv) $n + m^+ = (n + m)^+$

(iii) and (iv) help us in doing addition in N. Let us see the example for finding the result of addition of 3 and 4.

$3 + 4 = 3 + 3^+ = (3 + 3)^+ \ldots\ldots(1)$ [∵ of (iv)]

$3 + 3 = 3 + 2^+ = (3 + 2)^+ \ldots\ldots(2)$ [do]

$3 + 2 = 3 + 1^+ = (3 + 1)^+ \ldots\ldots(3)$ [do]

But $3 + 1 = 4$. [Known from (ii)]

Now, $(3) \Rightarrow 3 + 2 = (3 + 1)^+ = 4^+ = 5$

$(2) \Rightarrow 3 + 3 = (3 + 2)^+ = 5^+ = 6$

$(1) \Rightarrow 3 + 4 = (3 + 3)^+ = 6^+ = 7$

This conforms with the process we follow in the primary classes e.g.

$3 + 4 = 3 + (3 + 1)$

$= 3 + (2 + 1 + 1) = 3 + 2 + (1 + 1)$

$= 3 + (1 + 1) + (1 + 1) = 3 + (1 + 1 + 1 + 1)$

$= (3 + 1) + (1 + 1 + 1)$

$= 4 + (1 + 1 + 1)$

$= (4 + 1) + (1 + 1)$

$= 5 + (1 + 1)$

$= (5 + 1) + 1$

$= 6 + 1 = 7$

Multiplication in N

(v) $n \times 1 = n$ (vi) $n \times m^{+} = n \times m + n$

Multiplication in N is worked out following (v) and (vi) above. We want to multiply 5 and 3.

$5 \times 3 = 5 \times 2^{+} = 5 \times 2 + 5$ .....(1) [. of (vi)]

$5 \times 2 = 5 \times 1^{+} = 5 \times 1 + 5$ .....(2) [-do-]

$5 \times 1 = 5$ [∵ (v)]

Now, (2) $\Rightarrow$ $5 \times 2 = 5 \times 1 + 5 = 5 + 5 = 10$

(1) $5 \times 3 = 5 \times 2 + 5 = 10 + 5 = 15$

This also conforms with the process of multiplication followed in primary classes e.g.

$$\begin{aligned} 5 \times 3 &= 5 \times 2 + 5 \\ &= 5 \times 1 + 5 + 5 \\ &= 5 + 5 + 5 = 15 \end{aligned}$$

i.e. multiplication could be changed into continued addition.

Thus the processes of Addition and Multiplication made the Natural number set more useful to mankind.

Thereafter developed the processes of subtraction and division as the reverse processes of addition and multiplication respectively. Here of course the division had a restriction that a Natural number can only be divided by a factor of it.

Next comes the Eucledian alogrithem which states -

If $a, b, c, d \in N$, $b > d$ and

$a = b \times c + d$

Then we say that 'a' being divided by 'b' gives a result 'C' and leaves a remainder d. Now every natural

number can be divided by a natural number less than itself and in that case a remainder may or may not be left.

2.2 Some properties of Natural numbers:

Different natural numbers showed different behaviours with respect to division such as -

Some number are found to be divisible by only 2 numbers and we call them as Prime numbers and others (which are divisible by more than two numbers are known as Composite numbers. The table below shows the prime numbers (rounded of by circles) within the first 100 natural numbers.

1	2	3	4	5	6	7	8	9	0
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

It may be observed that -

the number of prime numbers within

1 to 50 = 15

the number of prime numbers within

51 to 100 = 10

The number of prime numbers within

1 to 1000 = 168

The number of prime numbers within

1000 to 2000 = 135.

It has been proved that prime numbers are infinitely many. 1 is a special number which is neither considered to be prime nor as composite.

There is an interesting property of the prime numbers. It is -

If 'p' is a prime number then 'p' divided by 6 leaves a remainder 1 or 5. Of course the converse is not true.

2.3 Inadequacy of Natural numbers:

'Necessity is the mother of invention', is a very important proverb which has played a vital role in the field of emergence of numbers.

Subtraction led to situation where Natural numbers were felt to be inadequate. For example -

$$4 - 4 = ?$$

No answer was available, since

$? + 4 = 4$ did not have any solution for us.

Hence a need for creating the number 0 (zero) was felt. Thus '0' was created. Further we had no answer for the problem.

$$3 - 5 = ?$$

$? + 5 = 3$ had no solution with us.

Thus the negative numbers -1, -2, -3 etc. emerged and the set of Integers Z came into existence.

Even and odd numbers in Z and their properties:

Integers divisible by 2 are even and the remaining integers are odd. Certain common properties are -

i) sum of two even integers is an even integer;

ii) product of two even integers is an even integer;

iii) sum of two odd integers is an odd integer;

iv) product of two odd integers is an odd integer;

v) 'a' is an even integer $\Rightarrow a^2$ is an even integer;

vi) 'a' is an odd integer $\Rightarrow a^2$ is an odd integer;

vii) a^2 is an even integer $\Rightarrow$ 'a' is an even integer;

viii) a^2 is an odd integer $\Rightarrow$ 'a' is an odd integer;

2.4 Inadequacy of Integers

Of course Eucledian algorithm gave us a method for dividing 'a' by 'b' when $a > b$, but we never got answer to the question -

how many 3 make 20 ?

i.e. $20 \div 3$ did not give a result so that there comes no remainder. Thus emerged the rational numbers :

Now we got an answer for the above division. It is

$$20 \div 3 = \frac{20}{3}$$

Thus we define -

$\frac{p}{q}$ is a rational number where $p, q \in Z$ and $q \neq 0$. Such numbers are known as rational numbers and the rational number set is denoted Q

2.5 Certain properties of rational numbers:

(i) Sum of two rational numbers is rational.

(ii) Product of two rational numbers is rational.

It can be shown that -

$x \in N \Rightarrow x \in Z$

and $x \in Z \Rightarrow x \in Q$

Thus $1 \in Q$, $0 \in Q$

iii) Division of a rational number by a non-zero rational number is also a rational number.

iv) There exist infinitely many rational numbers in between two distinct rational numbers.

Proof: Let $a, b \in Q$ and $a < b$,

$$a < b$$

$$\Rightarrow a + a < b + a$$

$$\Rightarrow 2a < a + b$$

$$\Rightarrow a < \frac{a+b}{2} \quad \ldots\ldots\ldots(1)$$

Again $a < b$

$$\Rightarrow a + b < b + b$$

$$\Rightarrow a + b < 2b$$

$$\Rightarrow \frac{a+b}{2} < b \quad \ldots\ldots\ldots(2)$$

(1) and (2) $\Rightarrow \quad a < \frac{a+b}{2} < b$

Sum of two rationals is a rational and quotient of two rationals with a non-zero denominator is a rational.

Thus in between 'a' and $\frac{a+b}{2}$ another rational number can also be available. So in between 'a' and 'b' infinitely many rationals can be made available by repeating this process.

2.6 Decimal form of rational numbers:

Following are certain examples which can be expressed in a form where the denominator takes the form of 10^n where $n \in N$ and such rationals are written in decimal form.

(A)

$$\frac{1}{10} = 0.1$$

$$\frac{2}{100} = 0.02$$

$$\frac{3}{5} = \frac{6}{10} = 0.6$$

$$\frac{11}{16} = \frac{11 \times 625}{16 \times 625} = \frac{6875}{10000} = 0.6875$$

It is obvious that rationals whose denominators have no other denominator other than 2 or 5, can be expressed in decimal forms which contains a specific number of digits after the decimal point. Let us take the rationals which are different from the ones discussed above and try to get the decimal forms by actual division.

(B)

$$\frac{1}{3} = 0.333 \ldots\ldots\ldots$$

$$\frac{5}{6} = 0.8333 \ldots\ldots$$

$$\frac{7}{15} = 0.4666 \ldots\ldots$$

$$\frac{233}{990} = 0.23535 \ldots.$$

The results obtained in decimal form (i) never come to an end (ii) certain digit (s) repeat endlessly.

Decimal form available from the fractions in Set 'A' are known as <u>terminating decimals</u>, whereas the decimal forms available from the fractions in Set 'B' are known as non-<u>terminating decimals</u>. Of course every terminating decimal can also be written as a nonterminating decimals as in the examples below.

$0.1 = 0.1000 \ldots\ldots$

$0.02 = 0.02000 \ldots\ldots$

$0.6 = 0.6000 \ldots\ldots$

Just by taking zeroes following the decimal digit a terminating decimal becomes a non terminating one.

Thus all the decimal numbers obtained in Set 'A' and 'B' both are non terminating decimals. In each one of those categories we find that some digit (s) repeat endlessly. For this property, such decimal numbers obtained from rationals are known as recurring decimals and those are written in a short form as shown below.

$0.1000 \ldots\ldots = 0.1\overline{0}$

$0.333 \ldots\ldots = 0.\overline{3}$

$0.23535 \ldots\ldots = 0.2\overline{35}$ and so on.

The part shown with a bar above is known as the repeating block.

It can also be proved that every recurring decimal number gives a rational number.

Now let us study the following decimal number.

0.123122312223 ....

It is definitely a non-terminating decimal But does it contain a repeating block ? A close study will help us answer the above question as 'no'. Hence it is not a rational number.

2.7 Inadequacy of rationals:

Integers provided solutions for all equations of the form $x + b = c$ where $b, c \in Z$. Rationals could give us solutions for equations of the form $ax + b = c$

What is the solution of $x^2 = C$ where $C \in Q$ Let us take certain examples.

$x^2 = 4 \Rightarrow x = \pm 2 \quad \because (+2)^3 = (-2)^2 = 4$

$x^2 = \frac{9}{16} \Rightarrow x = \pm \frac{3}{4} \quad \because (+\frac{3}{4})^2 = (-\frac{3}{4})^2 = \frac{9}{16}$

Can we get a solution for $x^2 = 2$? In other words can we get a rational number which being squared (multiplied by itself) gives 2 ?

Let us search for,

$(1)^2 = 1$ and $(2)^2 = 4$

But $1 < x^2 < 4$ [$\because x^2 = 2$ taken]

$\therefore \quad 1 < x < 2$

Let us now try with numbers lying between 1 and 2.

$(1.1)^2 = 1.12$ and $1.2 < 2$

$(1.2)^2 = 1.44$ and $1.44 < 2$

$(1.3)^2 = 1.69$ and $1.69 < 2$

$(1.4)^2 = 1.96$ and $1.96 < 2$

$(1.5)^2 = 2.25$ and $2.25 > 2$

$\therefore 1.4 < x < 1.5$

Now we try with numbers lying between 1.4 and 1.5

$(1.41)^2 = 1.9881$ and $1.9881 < 2$

$(1.42)^2 = 2.0164$ and $2.0164 > 2$

$\therefore 1.41 < x < 1.42$

Further continuing the search between 1.41 and 1.42 it observed that -

$(1.414)^2 = 1.999396$ and $1.999396 < 2$

$(1.415)^2 = 2.002225$ and $2.002225 > 2$

$\therefore 1.414 < x < 1.415$

In course of the search in the manner as indicated above men must have consumed a good deal of their time and energy and would still have arriving at the number they intended to get in their rational domain.

Then he must have tried to see logically of at all any such rational is possible to give the solution of the equation $x^2 = 2$. His happiness must have gone beyond bounds when he could logically prove that no rational solution is available for the above equation.

Then a new kind of number emerged which were named as irrationals (which can not be expressed in the form $\frac{p}{q}$ where $p, q \in Z$ and $q \neq 0$.

$\sqrt{2}$ was taken as the solution for representing x such that $x^2 = 2$. Similarly $\sqrt{3}$ is the solution for $x^2 = 3$ and so on.

In the process of search as illustrated above or by the process of determining square root we may continue to determine the decimal value of $\sqrt{2}$, it will never come to an end and whatsoever we get is just a <u>rational approximation</u> of $\sqrt{2}$. Further it can be seen that there will be no repeating block in the rational approximation of $\sqrt{2}$ determined to any number of digits.

2.8 <u>Square root of n and $\sqrt{n}$ [where $n \in Z^+$]</u>

Square root of a number is a number whose square is equal to the given number. Thus,

Square root of 4 = $\pm$ 2 [$\because (+2)^2 = (-2)^2 = 4$]

Square root of 9 = $\pm$ 3 [$\because (+3)^2 = (-3)^2 = 9$]

Thus it is observed that the square root of a perfect square number n ($n \in Z^+$) are two opposite integers (additive inverse) and both of them are available by using one numeral i.e. 2 in case of square root of 4.

Now let us think of the equation $x^2 = 2$ again.

Is $\sqrt{2}$ the only solution ?

Since $-\sqrt{2}$ is the opposite number to $\sqrt{2}$, $-\sqrt{2}$ is also a solution. Thus -

$x^2 = 2$ has two solutions and that are $\sqrt{2}$ and $-\sqrt{2}$.

$x^2 = 3$ has two solutions and that are $\sqrt{3}$ and $-\sqrt{3}$.

In the same analogy we say -

$x^2 = 4$ has two solutions and that are

$\sqrt{4}$ and $-\sqrt{4}$ which are 2 and - 2 respectively.

As such $\sqrt{4} = 2$ and $-\sqrt{4} = -2$

Hence $\sqrt{4}$ gives the positive square root of 4 which is 2 where as square root of 4 are 2 and -2 as well.

2.9 <u>A different kind of irrationals:</u>

The irrationals which have already been talked of were the solutions of some or other algebraic equation, such as -

$\sqrt{2}$ is the solution of $x^2 = 2$

$\sqrt[3]{4}$ is the solution of $x^3 = 4$ and so on.

There is another category of irrationals which do not emerge as solutions of algebraic equations. These are -

log 2, log 3 - - - -, -

$$\pi = \frac{\text{circumference of a circle}}{\text{its diameter}}$$

Sin 20 etc.

There are non algebraic irrationals.

All the irrationals (algebraic or nonalgebraic irrationals taken together) are denoted as Q'.

2.10 Real number system:

The rationals and irrationals taken together constitute the real number set (denoted as R). Thus -

$$Q \cup Q' = R$$

To bring in an association between Algebra and Geometry one - one correspondence has been established between a line L and the set of Real numbers R.

Thus $L \leftrightarrow R$

That is for every point on a line there exists a real number and for every real number. There exists a point on a line.

2.11 Operations in R

Addition and multiplications are two operations in R. Various property of the operations are as follows:

Properties	Addition	Multiplication
1. Closure	$a,b \in R \Rightarrow a+b \in R$	$a,b \in R \Rightarrow a.b \in R$
2. Commutative	$a,b \in R \Rightarrow a+b = b+a$	$a,b \in R \Rightarrow a.b = b.a$
3. Associative	$a,b,c \in R \Rightarrow a+(b+c) = (a+b)+c$	$a,b,c \in R \Rightarrow a(b.c) = (a.b)c$
4. Existence of identity	$a \in R \Rightarrow a+0 = 0+a = a$ [0 is the additive identity]	$a \in R \Rightarrow a.1 = 1.a = a$ [1 is the multiplicative identity]
5. Existence of inverse	$a \in R \Rightarrow -a \in R$ such that $a+(-a)=0$	$a \in R \Rightarrow \frac{1}{a} \in R$ such that $a \times \frac{1}{a} = 1$
6. Distributive (Addition & multiplication)	$a,b,c \in R \Rightarrow a(b+c) = ab + ac$	

S U R D

3.13 Introduction:

If a is any positive rational number which can not be expressed as the n^{th} power of some rational number then the irrational number $a^{\frac{1}{n}}$ or $\sqrt[n]{a}$ which as the positive n^{th} root of a 'a' is called a surd or a radical.

Example: $2^{\frac{1}{2}}, 2^{\frac{1}{3}}, 3^{\frac{1}{2}}, 5^{\frac{1}{2}}, 5^{\frac{1}{3}}, 5^{\frac{1}{4}}, 21^{1/5}$ etc.

the examples of surds.

$[\because 2 \neq n^2$ where $n \notin Q$, $\therefore 2^{1/2}$ is a surd

In $\sqrt[n]{a}$ surd the symbol $\sqrt[n]{\ }$ is called as the radical sign, n is called as the order of the surd or simply radical and a is called as the radicand.

So from the definition of the surd of the type $\sqrt[n]{a}$, it is clearly understood that a is a rational number and $\sqrt[n]{a}$ is an irrational number, $\sqrt{\sqrt{2}}$ appears to be not a surd because it is the Square root of an irrational number ($\sqrt{2}$), but it is considered as a surd because it can be expressed as $\sqrt[4]{2}$. $\sqrt{5}$ is of order 2, $\sqrt[3]{5}$ is of order 3. $\sqrt{4}$ is not a surd since $\sqrt{4} = 2$ which is rational.

$3 + 2\sqrt{2}$ is not a surd since $3 + \sqrt{2}$ is not a rational number.

3.14 Laws of radicals

1. For any positive integer n and a positive rational number a the radical $\sqrt[n]{a}$ is the positive n^{th} root of a $\Rightarrow (\sqrt[n]{a})^n = a$.

2. $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are two radicals of the same order then

$$(\sqrt[n]{a} \cdot \sqrt[n]{b})^n = (\sqrt[n]{a})^n (\sqrt[n]{b})^n$$

$$\Rightarrow \sqrt[n]{a}\ \sqrt[n]{b} = \sqrt[n]{ab}$$

3. If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are two radicals of the same order then

$$\left(\frac{\sqrt[n]{a}}{\sqrt[n]{b}}\right)^n = \frac{(\sqrt[n]{a})^n}{(\sqrt[n]{b})^n} = \frac{a}{b}$$

$$\Rightarrow \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

4. If m, n are positive integers then for a positive rational number. a

$$\left[\sqrt[m]{(\sqrt[n]{a})}\right]^{mn} = (\sqrt[n]{a})^n = a$$

similarly $\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$

$$\Rightarrow \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$$

5. If m, n are positive integers, then for a positive rational number a

$$\sqrt[n]{a^p} = \sqrt[n]{\sqrt[m]{(a^p)^m}} = \sqrt[mn]{a^{pm}}$$

The index of the radical and exponent of the radicand one both multiplied by the same number m.

Examples: $(\sqrt[3]{7})^3 = 7$

2. $\sqrt{125} = \sqrt{25 \times 5} = \sqrt{25}\sqrt{5} = 5\sqrt{5}$

3. $\sqrt[3]{\frac{2}{3}} = \frac{\sqrt[3]{2}}{\sqrt[3]{3}}$

4. $\sqrt[4]{64} = \sqrt[2 \times 2]{64} = \sqrt[2]{\sqrt[2]{64}}$

$= \sqrt{8}$

5. $\sqrt[6]{64} = \sqrt[2 \times 3]{64} = \sqrt{4} = 2$

6. $\sqrt[5]{\sqrt[3]{7}} = \sqrt[5 \times 3]{7} \quad \sqrt[15]{7}$

A surd in its simplest form has

i) no factor which is n^{th} power of a rational number under the radical sign whose index is n .

ii) no fraction under the radical sign.

iii) the smallest positive index of this radical

Simplify :- $\sqrt{\frac{250}{63}}$

$$\sqrt{\frac{250}{63}} = \frac{\sqrt{250}}{\sqrt{63}} = \frac{\sqrt{2 \times 5 \times 5 \times 5}}{\sqrt{3 \times 3 \times 7}} =$$

$$\frac{5\sqrt{10}}{3\sqrt{7}} = \frac{5\sqrt{10}\sqrt{7}}{3\sqrt{7}\sqrt{7}}$$

$$= \frac{5\sqrt{70}}{3 \times 7} = \frac{5\sqrt{70}}{21}$$

3.15 Mixid Surd:

A surd which has a rational factor other than unity, the other factor being irrational is called a mixid surd.

3.16 Pure Surd:

A surd which has unity as its rational factor, the other factor being irrational is called a pure surd.

$\sqrt{5}$, $\sqrt{7}$, $\sqrt[3]{5}$ are pure surds

$2\sqrt{3}$, $5\sqrt[3]{18}$, $3\sqrt[3]{2}$ are mixed surds .

The laws of radicals enables us to express a pure surd as a mixed surd or vice versa.

Expression of (i) $\frac{2}{3}\sqrt{32}$ as a pure surd

$\frac{2}{3}\sqrt{32} = \sqrt{\left(\frac{2}{3}\right)^2}\sqrt{32} = \sqrt{\left(\frac{2}{3}\right)^2 \times 32}$

$= \sqrt{\frac{4}{9} \times 32} = \sqrt{\frac{128}{9}}$

ii) $2\sqrt[3]{4} = (2^3)^{1/3}\sqrt[3]{4} = \sqrt[3]{2^3}\sqrt[3]{4}$

$= \sqrt[3]{2^3 \cdot 4} = \sqrt[3]{8.4} = \sqrt[3]{32}$

Expression of as mixed surd

i) $\sqrt[3]{72} = \sqrt[3]{2^3 . 3^2} = \sqrt[3]{2^3}\sqrt[3]{3^2}$

$= 2\sqrt[3]{3^2} = 2\sqrt[3]{9}$

ii) $5\sqrt[3]{135} = 5\sqrt[3]{5.3^3} = 5\sqrt[3]{5}\sqrt[3]{3^3}$

$= 5\sqrt[3]{5}. 3 = 15\sqrt[3]{5}$

3.17 <u>Comparison of Surds:</u>

If the surds are of same order then the surds are compared according to their radicands. If the surds are of different orders then at first all have to be reduced to the same order and then compare according to their radicands .

Example: (ii) $\sqrt[3]{2} < \sqrt[3]{3} < \sqrt[3]{5} < \sqrt[3]{7}$

Since they haoe of same order (3) and $2 < 3 < 5 < 7$

(iii) of $\sqrt[3]{7}$, $\sqrt[4]{5}$ which one is greater ? First take the L.C.M. of 3 and 4 which is 12

$\Rightarrow \sqrt[3]{7} = \sqrt[3 \times 4]{7^4} = \sqrt[12]{2401}$

$\sqrt[4]{5} = \sqrt[4 \times 3]{5^3} = \sqrt[12]{125}$

Since they have equal order and

$125 < 2401 \Rightarrow 4\sqrt{5} < 3\sqrt{7}$

3.18 Addition and subtraction of Surds:

Surds having the same erratıonal factor are called as sımılar surds. Using the dıstributing law similar surds can be added and subtracted.

Simplyfies:- $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

Reducing each into the sımplest form

$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

$3\sqrt{20} = 3\sqrt{4 \times 5} = 3 \times 2\sqrt{5} = 6\sqrt{5}$

The given numerical expression

$= \sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5}$

$= (3 - 6 + 4)\sqrt{5} = \sqrt{5}$

3.19 Rationalisation of Surds

When the product of two surds in a rational number then each of the two surds ıs called Rationalısıng factor of the other.

(i) $3\sqrt{7} \times \sqrt{7} = 21$

So 7 is a R.F. of $3\sqrt{7}$

(ii) $4\sqrt{7} \times \sqrt{7} = 28$

So 7 is a R.F. of $4\sqrt{7}$

Therefore rationalısıng factor is not unique.

(iii) $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = 5 - 3 = 2$

$\sqrt{5} + \sqrt{3}$ is R.F. of $\sqrt{5} - \sqrt{3}$ and vice versa

(iv) $(\sqrt{7}+\sqrt{5}-\sqrt{3})(\sqrt{7}+\sqrt{5}+\sqrt{3})$

$\Rightarrow (\sqrt{7}+\sqrt{5})^2-(-\sqrt{3})^2 = (\sqrt{7}+\sqrt{5})^2 - 3$

$= 7+5+2\sqrt{35}-3 = 9+2\sqrt{35}$

$(9+2\sqrt{35})(9-2\sqrt{35}) = 9^2-(2\sqrt{35})^2$

$= 81 - 140 = -59$ which is rational .

$\Rightarrow (\sqrt{7}+\sqrt{5}-\sqrt{3})(\sqrt{7}+\sqrt{5}+\sqrt{3})$

$(9-2\sqrt{35}) = -59$

$\Rightarrow (\sqrt{7}+\sqrt{5}+\sqrt{3})(9-2\sqrt{35})$ is R.F. of

$\sqrt{7}+\sqrt{5}-\sqrt{3}$

3.20 Rationalisation of monomial Surds:

(1) $\sqrt{32} = \sqrt{2^5} = \sqrt{2^4 \times 2} = 2^2\sqrt{2} = 4\sqrt{2}$

(2) $\sqrt[3]{72} = \sqrt[3]{2^3 \times 3^2} = 2\sqrt[3]{3^2} = 2\sqrt[3]{9}$

From the smallest rationalising factor

so that $\sqrt{32}$ is rational, since $\sqrt{32} = 4\sqrt{2}$

Since $4\sqrt{2} \times \sqrt{2} = 4 \times 2 = 8$

So $\sqrt{2}$ is the smallest rationalising factor

of $4\sqrt{2}$ i.e. $\sqrt{32}$

Similarly $\sqrt[4]{81} - 3\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

Reducing to each of the terms into

simplest forms

$\sqrt[4]{81} = \sqrt[4]{3^4} = 3$

$8\sqrt[3]{216} = 8\sqrt[3]{6^3} = 8 \times 6 = 48$

$15\sqrt[5]{32} = 15\sqrt[5]{2^5} = 15 \times 2 = 30$

$\sqrt{225} = \sqrt{3^2 \times 5^2} = 3 \times 5 = 15$

The given numerical expression:

$= \sqrt[4]{81} - 3\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

$= 3 - 48 + 30 + 15 = 0$

Simplify $\sqrt{75} + \frac{4}{\sqrt{3}}$

$$\sqrt{75} + \frac{4}{\sqrt{3}} = \sqrt{5^2 \times 3} + \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 5\sqrt{3} + \frac{4\sqrt{3}}{3} = \left(5 + \frac{4}{3}\right)\sqrt{3} = \frac{19}{3}\sqrt{3}$$

3.21 <u>Multiplication and division of two Surds:</u>

The surds of the same order will be multiplied according to the law given below.

$$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$$

If the surds are not in the same order then we have to reduce them into the same order first, then apply the law given above.

Simplify and express in its simplest form.

(i) $\sqrt{28} \times \sqrt{21} = \sqrt{23 \times 21} = \sqrt{2^2 \times 7 \times 7 \times 3} =$

$2 \times 7\sqrt{3} = 14\sqrt{3}$

(ii) $\sqrt[3]{2} \times \sqrt{5} = \sqrt[3 \times 2]{2^2} \times \sqrt[3 \times 2]{5^3}$

$= \sqrt[6]{4} \times \sqrt[6]{125} = \sqrt[6]{4 \times 125} = \sqrt[6]{500}$

(iii) Divide $4\sqrt{28}$ by $3\sqrt{7}$

$$\frac{4\sqrt{28}}{3\sqrt{7}} = \frac{4\sqrt{2^2 \times 7}}{3\sqrt{7}} = \frac{4 \times 2\sqrt{7}}{3\sqrt{7}} = \frac{8}{3}$$

(iv) $\sqrt[4]{12} \div \sqrt[4]{3} = \sqrt[4]{\frac{12}{3}} = \sqrt[4]{4}$

(v) $\sqrt{24} \div \sqrt[3]{200} = \sqrt[6]{24^3} \div \sqrt[6]{200^2} = \sqrt[6]{\frac{24^3}{200^2}}$

$= \sqrt[6]{\frac{13824}{40000}} = \sqrt[6]{\frac{216}{625}}$

(vi) $\sqrt[3]{72} = 2\sqrt[3]{3^2} = 2.3^{2/3}$

$2.3^{2/3},\ 3^{1/3} = 2.3 = 6$

$3^{1/3} = \sqrt[3]{3}$ is the smallest or simplest rationalising factor of $\sqrt[3]{72}$

(3) Find the rationalising factor (R.F.) of $\sqrt[4]{a^2 b^3 c^4}$ where a, b, c are rational numbers.

$\sqrt[4]{a^2 b^3 c^4} = a^{2/4}, b^{3/4}, c = a.^{1/2} b^{3/4}. c$

Since $(a^{1/2}. b^{3/4}. c)(a^{1/2}. b^{1/4}) = abc$

$a^{1/2} b^{1/4}$ is a R.F. of $\sqrt[4]{a^2 b^3 c^4}$

$\sqrt[4]{a^2 b}$ is a R.F. of $\sqrt[4]{a^2 b^3 c^4}$

Surds of second order are called as quadratic surds.

The simplest R.F. of binomial quadratic surds is its conjugate surd.

Since $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = x - y$

Which is rational since one is the R.F. of the other.

$(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = 5 - 3 = 2$

$(3 + \sqrt{5})(3 - \sqrt{5}) = 9 - 5 = 4$

$3 + \sqrt{5}$ is the conjugate of $3 - \sqrt{5}$ and $3 - \sqrt{5}$ is the conjugate of $3 + \sqrt{5}$

$\sqrt{5} + \sqrt{3}$ is the conjugate of $\sqrt{5} - \sqrt{3}$ and $\sqrt{5} - \sqrt{3}$ is the conjugaee of $\sqrt{5} + \sqrt{3}$.

Express $\dfrac{3\sqrt{2}}{\sqrt{6} - \sqrt{3}}$ with a rational denominator

$$\frac{3\sqrt{2}}{\sqrt{6} - \sqrt{3}} = 3\,\frac{\sqrt{2}(\sqrt{6} + \sqrt{3})}{(\sqrt{6} - \sqrt{3})(\sqrt{6} + \sqrt{3})} - \frac{3\sqrt{2}(\sqrt{6} + \sqrt{3})}{6 - 3} =$$

$$\frac{3\sqrt{2}(\sqrt{6} + \sqrt{3})}{3} = \sqrt{2}(\sqrt{6} + \sqrt{3}) = \sqrt{12} + \sqrt{6}$$

Simplify $\dfrac{\sqrt{3} + \sqrt{5}}{\sqrt{5} - \sqrt{3}} + \dfrac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$

$$\frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}-\sqrt{3}} = \frac{(\sqrt{5}+\sqrt{3})\ (\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})\ (\sqrt{5}+\sqrt{3})} = \frac{5+3+2\sqrt{15}}{5-3}$$

$$= \frac{8+2\sqrt{15}}{2} = 4+\sqrt{15}$$

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}-\sqrt{3})\ (\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})\ (\sqrt{5}\ \sqrt{3})} = \frac{5+3-2\sqrt{15}}{5-3}$$

$$= \frac{8-2\sqrt{15}}{2} = 4+\sqrt{15}$$

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}-\sqrt{3})\ (\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})\ (\sqrt{5}-\sqrt{3})} = \frac{5+3-2\sqrt{15}}{5-3}$$

$$= \frac{8-2\sqrt{15}}{2} = 4-\sqrt{15}$$

$$\frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = (4+\sqrt{15}) + (4-\sqrt{15}) = 8$$

N.3. $\frac{m}{n}$ is possible if $a < 0$ provided $\frac{m}{n}$ is expressed in the lowest form and n is odd.

Exp. $(-8)^{1/3} = -2$

FUNCTION

4.1 Introduction:

Before going into the topic let us discuss about relations. Let A and B be two non-empty sets then

$A \times B = \{(x,y) / x \in A,\ y \in B\}$ is the cartesian product of A and B.

Let $A = \{1, 2\}$, $B = \{1, 4, 9\}$

$\Rightarrow A \times B = \{(1, 1), (1, 4), (1,9), (2,1), (2,4), (2, 9)\}$

It can be seen that $n(A \times B) = n(A) \times n(B)$.

Any subset of $A \times B$ is a relation from A into B.

In this example there are 2^6 sub-sets of $A \times B$.

So we can find 2^6 relations from A into B.

The relations are $\{(1,1)\}$, $\{(1,1), (1,9)\}$, $\{(2,1), (2, 4), (2, 9)\}$ etc.

The 1st component of an ordered pair contained in a relation is known as the pre-image and the 2nd component is known as the image. Thus in (a, b) 'a' is the pre-image and 'b' is the image .

We come across four types of relations they are (i) one - one (ii) Many one (iii) Many - many and (iv) One - many relations.

(i) One - One relation.

When one element of A is related to one and only one element of B then the relation is said to be one - one.

Example:- $A = \{1,2,3\}$, $B = \{1, 4, 9, 16\}$

$R_1 = \{(1,1)\}$, $R_2 = \{(1,4)\}$,

$R_3 = \{(1,9)\}$, $R_4 = \{(1,16)\}$

$R_5 = \{(1,1), (2,9)\}$, $R_6 = \{(1,16),$

$(2,9), (3, 4)\}$ etc.

We also say - In a one - one relation, for each pre image of the ordered pair, contained in the relation, there exists an unique image.

ii) Many-One relation:-

When one element of A is related to more than one element of B then the relation is said to be many one relation.

Example:-

$R_7 = \{(1,1), (2,1)\}$, $R_8 = \{(2,4),$
$(3,4), (3, 16)\}$.

We also say - In a many-one relation, more than one pre-image exists for the same image of the ordered pairs contained in the relation.

iii) Many-many relation:- When more than one element of A are related to more than one element of B, then the relation is many - many relation.

Example:- $R_9 = \{(1,1), (1,4), (2,9), (2,16),$
$(3, 9)\}$.

We also say - In a many - many relation more than one pre-images have the same image in certain ordered pairs and the same pre-image has more than one images in certain ordered pairs contained in the relation.

iv) One - Many relation:-

When one element of A is related to more than one element of B then the relation is called as one - many relation.

Example:-

$R_{10} = \{(1,1), (1,4), (2,16), (2,19)\}$ etc.

We also say - In a one - many relation, one preimage corresponds to more than one images in certain ordered pairs contained in the relation.

Domain of the relation R is denoted as D (R).

$\Rightarrow D(R) = \{x \in A \,/\, (x,y) \in R\}$

Similarly range of the relation R is denoted as

$R(R) = \{y \in B \mid (x,y) \in R\}$

Example:-

$R_1 = \{(1,1)\}$, $D(R_1) = \{1\}$, $R(R) = \{1\}$,

$D(R_8) = \{2, 3\}$, $R(R_8) = \{4, 16\}$.

One of these four types of relations only two of these may be functions. They are one - one and many - one provided the domain of the relation is the same as the set A. So f is a function from A into B if

(i) $f \subset A \times B$ is (f is a relation)

(ii) $D(f) = A$

(iii) If $(x, y_1) \in f$ and $(x, y_2) \in f \Rightarrow y_1 = y_2$

(uniqueness of the image)

For the function the relation must not be many-many and one - many. So f is a function from A into B if $f \subseteq A \times B$ and for every $x \in A$ there exists a unique element $y \in B$ such that $(x,y) \in f$. Thus symbolically represented as

$f : A \longrightarrow B$

Such that $f(x) = y$

x is called as the pre image of y.

y is called as the image of x

A is called as the domain of f

B is called as the codomain of f

$\{f(x) \;/\; x \in A\}$ is called as the range of f.

Example:- $A = \{1,2,3,4,5\}$, $B = \{1,4,9,16,25,36\}$

$f : A \longrightarrow B$ such that $f(x) = x^2$

$f(1) = 1^2 = 1$

$f(2) = 2^2 = 4$

$f(3) = 3^2 = 9$

$f(4) = 4^2 = 16$

$f(5) = 5^2 = 25$

$D(f) = \{1,2,3,4,5\} = A$

$R(f) = \{1,4,9,16,25\}$

Codomain of $f = B = \{1,4,9,16,25,36\}$

4.1 The graph of a real function:

Let $S \subseteq R$ where R is the set of real number. and $f : S \longrightarrow R$ is called a real function.

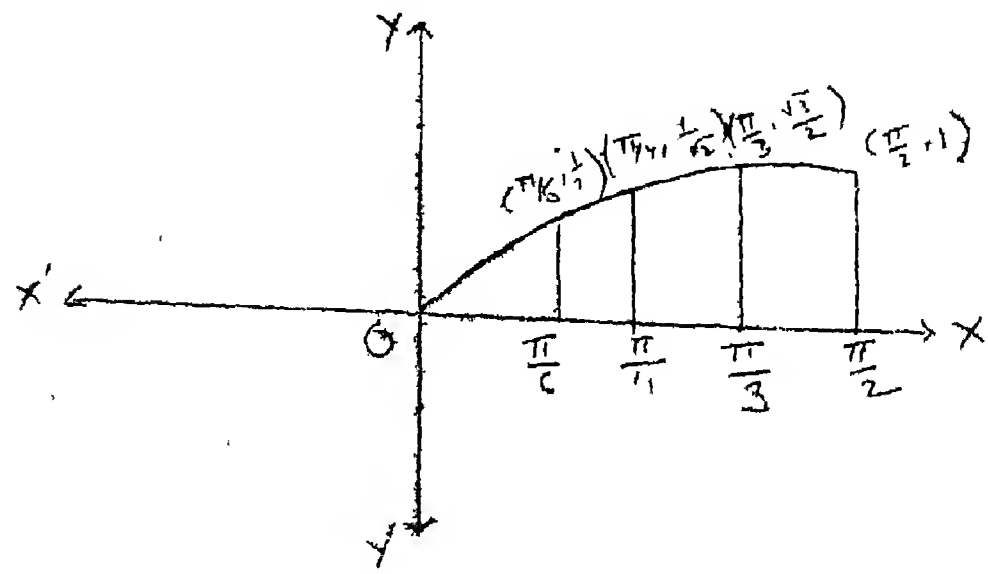
In the real function we denote the image of x under f as y which is written as $y = f(x)$

x is called as the independent variable and y is called as the dependent variable.

We can write some of the values of the dependent and independent variable and plot the graph of the function.

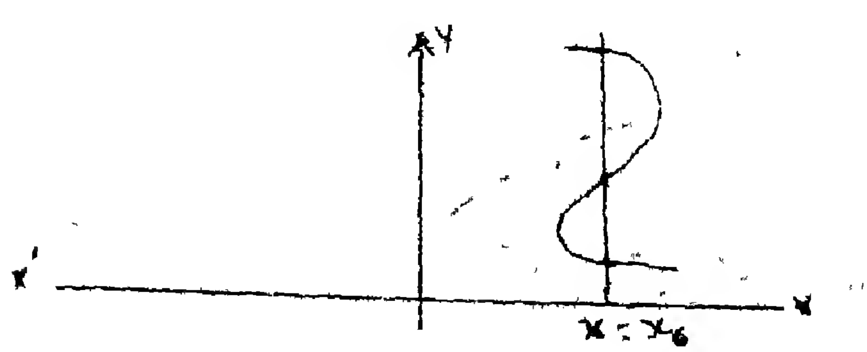
Example: $y = \sin x$

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
y = Sin x	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1



Note:- Sin x is a function since for all ordered pairs (x, Sin x), $x \in R$ and Sin x is unique for every x.

The graph of a relation as indicated below shows that the image corresponding to $x = x_0$ is not unique . As a result the relation y is not a function.



For the function the relation must not be many-many and one - many. So f is a function from A **into** B if $f \subset A \times B$ and for every $x \in A$ there exists a unique element $y \in B$ such that $(x, y) \in f$. Thus symbolically represented as

$f : A \longrightarrow B$

Such that $f(x) = y$

x is called as the pre image of y.

y is called as the image of x

A is called as the domain of f

B is called as the codomain of f

$\{f(x) / x \in A\}$ is called as the range of f.

Example:- $A = \{1,2,3,4,5\}$, $B = \{1,4, 9,16,25,36\}$

$f : A \longrightarrow B$ such that $f(x) = x^2$

$f(1) = 1^2 = 1$	
$f(2) = 2^2 = 4$	$D(f) = \{1,2,3,4,5\} = A$
$f(3) = 3^2 = 9$	$R(f) = \{1,4,9,16,25\}$
$f(4) = 4^2 = 16$	Codomain of $f = B =$
$f(5) = 5^2 = 25$	$\{1,4,9,16,25, 36\}$

4.1 <u>The graph of a real function:</u>

Let $S \subset R$ where R is the set of real number. and $f : S \longrightarrow R$ is called a real function.

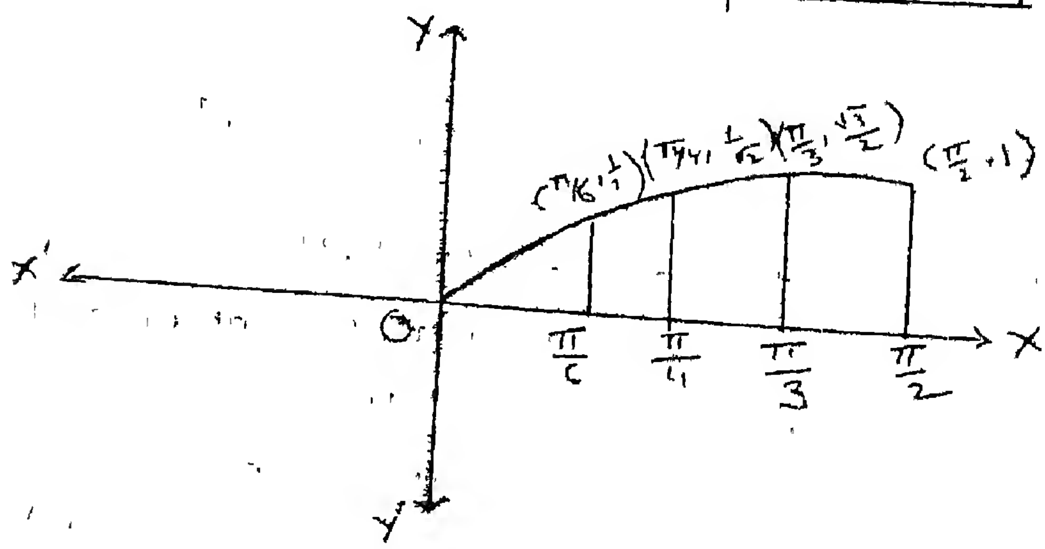
In the real function we denote the image of x under f as y which is written as $y = f(x)$

x is called as the independent variable and y is called as the dependent variable.

We can write some of the values of the dependent and independent variable and plot the graph of the function.

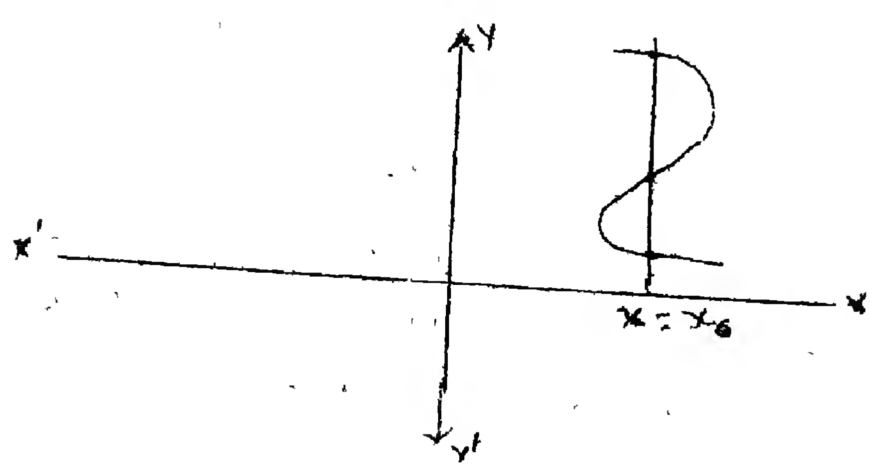
Example: $y = \sin x$

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$y = \sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

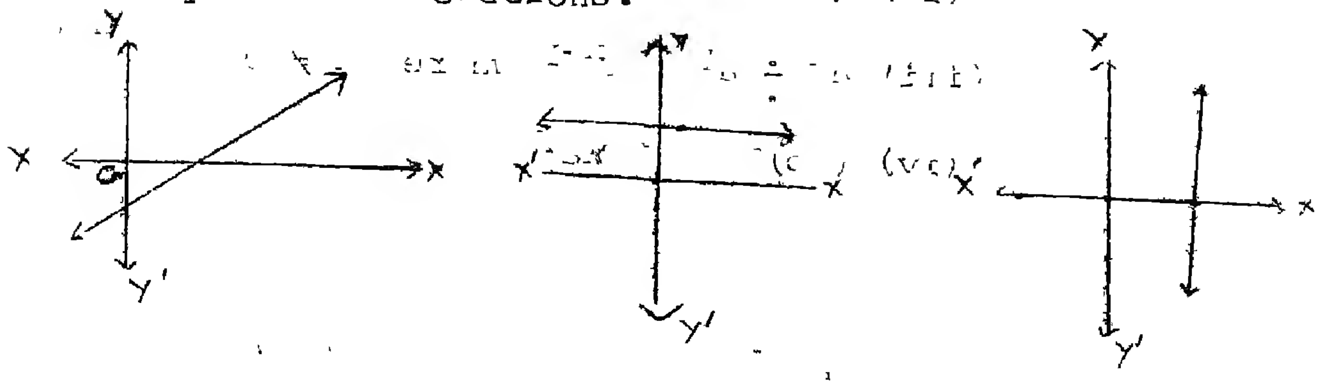


Note:- Sin x is a function since for all ordered pairs (x, Sin x), $x \in R$ and Sin x is unique for every x.

The graph of a relation as indicated below shows that the image corresponding to $x = x_0$ is not unique . As a result the relation y is not a function.

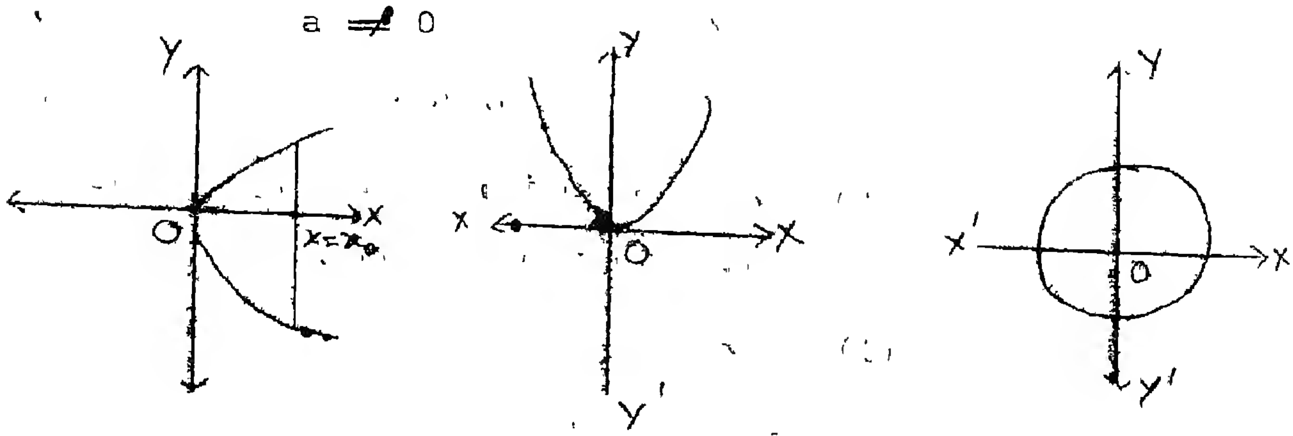


Let us study the following graphs represented by various Relations.



(i) $y = ax + b$ (ii) $y = c$ $x = K$

$a \neq 0$



(iv) $y^2 = x$ (v) $y = x^2$ (v) $x^2+y^2 = r^2$

Of the 6 graphs shown above, the relation representing the graph at (i), (ii), (v) are functions where as the relations representing the graphs at (iii), (iv) and (vi) are not functions. Since images are not unique.

5.1 Indices and Logarithm

Introduction:

Product of repeated factors like '$3 \times 3 \times 3 \times 3 \times 3$' is also written in the form 3^5.

In 3^5, 3 is known as the base and 5 is known as the <u>index</u> or <u>exponent</u>. Plural of the term index is <u>indices</u>. Thus the

discussion relating to index goes under the title , 'Indices '.

We start discussing about logarithm with the following definition. For any real number 'a' and a positive integer 'n', we define a^n as -

$a^n = a \times a \times a \times \ldots\ldots$ up to n factors.

The following rules can be proved.

(i) $a^m \times a^n = a^{m+n}$ $\left[\text{where } a \in R,\ m,\ n \in N\right]$

(ii) $(a^m)^n = a^{mn}$ -do-

(iii) $a^m \div a^n = a^{m-n}$ $\left[\text{where } a \in R\ \ a \neq 0\ m,n \in N\right]$ and $m > n$.

We also define :-

$a^0 = 1$ where $a \in R$ and $a \neq 0$

$a^{-n} = \frac{1}{a^n}$ where $a \in R,\ a \neq 0,\ n \in N$

$a^{\frac{1}{n}} = \sqrt[n]{a}$ where $a \in R,\ a > 0\ ,\ n \in N$

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ or $\sqrt[n]{a^m}$ where $a \in R$, $a > 0\ m,\ n \in Z,\ n \neq 0$

Taking a, b as positive and $x, y \in R$ we accept the following axioms.

(i) $a^x \times a^y = a^{xy}$

(ii) $(a^x)^y = a^{xy}$

(iii) $a^x \div a^y = a^{x-y}$ where $a \neq 0$

(iv) $(ab)^x = a^x \times a^y$

From the above discussions and definitions it is clear that the meaning of a^n has different meanings in different situations. There are certain limitations for 'a' and 'x' which are indicated in the definitions.

<u>Some examples:</u>

i) $5^0 = 1$ ii) $8^{-2} = \frac{1}{8^2} = \frac{1}{64}$

iii) $\sqrt{x} = x^{\frac{1}{2}}$, $x > 0$

iv) $x^{-2} = \frac{1}{x^2}$ where $x \neq 0$

v) $\sqrt{3} = 3^{\frac{1}{2}}$

Logarithm

5.2 Definition:

If for a positive real number $a (a \neq 1)$

$$a^m = b$$

We say that -

m is the logarithm of b to the base a.

We write this as $\log_a b = m$.

That is $a^m = b \Leftrightarrow \log_a b = m$

Thus logarithmic statements and exponential statement are interchangeable. Examples follows.

(i) $3^4 = 81 \Leftrightarrow \text{Log}_3 81 = 4$

(ii) $2^5 = 32 \Leftrightarrow \log_2 32 = 5$

(iii) $\log_{10} 100 = 2 \Leftrightarrow 10^2 = 100$

(iv) $\log_3 x = y \Leftrightarrow 3^y = x$

Laws of logarithm

If x, y and a are positive real numbers and $a \neq 1$, then

(i) $\log_a (x\,y) = \log_a x + \log_a y$

(ii) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

(iii) $\log_a x^y = y \log_a x$

5.3 Usefulness of logarithm:

The laws of logarithm show that logarithm changes -

(i) The operation of multiplication to addition such as $\log_a xy = \log_a x + \log_a y$

(ii) The operation of division to subtraction such as $\log_a \frac{x}{y} = \log_a x - \log_a y$

(iii) Exponential operation to product such as $\log x^y = y \log x$

Thus logarithm makes the numerical computation easier.

5.4 Common logarithm:

Logarithm with base 10 is known as Common logarithm. The following are the examples.

$10^0 = 1 \Leftrightarrow \log_{10} 1 = 0$

$10^1 = 10 \Leftrightarrow \log_{10} 10 = 1$

$10^2 = 100 \Leftrightarrow \log_{10} 100 = 2$

$10^5 = 100000 \Leftrightarrow \log_{10} 100000 = 5$

N.B. logarithms reduces very large numbers into quite small ones.

Again –

$$10^{-1} = \frac{1}{10} = 0.1 \Leftrightarrow \log 0.1 = -1$$

$$10^{-2} = \frac{1}{100} = 0.01 \Leftrightarrow \log 0.01 = -2$$

$$10^{-4} = \frac{1}{10000} = 0.0001 \Leftrightarrow \log 0.0001 = -4$$

Further it is seen that logarithm changes fractions of much higher order to integers of comparitively much smaller order.

5.5 Characteristic and Mantissa in Common logarithm

We have seen that

$\log 1 = 0$ and $\log 10 = 1$

What about $\log n$ when $1 < n < 10$?

Obviously

$$\log 1 < \log n < \log 10$$

$$\Rightarrow 0 < \log n < 1$$

$$\log n = 0 + m \text{ where } 0 \leq m < 1$$

that is, m is a +ve fraction.

Similarly –

when $10 < n < 100$

$$\log 10 < \log n < \log 100$$

$$\Rightarrow 1 < \log n < 2$$

$$\Rightarrow \log n = 1 + m \text{ where } 0 < m < 1$$

when $0.1 < n < 1$

$$\log 0.1 < \log n < \log 1$$

$$\Rightarrow -1 < \log n < 0$$

$$\Rightarrow \log n = -1 + m \text{ where } 0 < m < 1$$

Thus $\log n = c + m$

where $c \in Z$ and $0 < m < 1$

c is known as the <u>characteristic</u> and m is known as the <u>mantissa</u>.

5.6 Standard form of writing rational numbers:

For convenience we follow a certain method of writing rational numbers which is illustrated below.

$$4357.982 = 4.357982 \times 10^{3}$$

$$0.0000327 = 3.27 \times 10^{-5}$$

Each of the two numbers given above has been written in standard form. In general we can say that -

$$n = a \times 10^{c}$$

where $n \in Q$, $a \in Q$, $1 < a < 10$ and $c \in Z$.

Taking logarithm of both the sides we get -

$$\log n = \log (a \times 10)^{c}$$

$$\Rightarrow \log n = \log a + \log 10^{c}$$

$$\Rightarrow \log n = \log a + c \times \log 10$$

$$\Rightarrow \log n = \log a + c$$

$$\Rightarrow \log n = c + \log a$$

Thus expressing the given number 'n' in standard form the characteristic and mantissa of log n can be determined. The mantissa is always a positive decimal number.

5.7 Determination of mantissa

A table, known as log. table provides us the approximate values of log a where $1 < a < 10$. The table which the secondary school students follow is a four figure table i.e. for all 'a' lying between 1 and 10 and having 4 digits, the value of log a can be obtained from it. There are tables for more number of figures (digits) also.

Alternative rules for determining characteristic

The following examples will show an alternative rule for determining the characteristic.

$$\log 23.79 = \log (2.379 \times 10^1)$$

$$= 1 + \log 2.371$$

$$\log 350.8 = \log (3.508 \times 10^2)$$

$$= 2 + \log 3.508$$

Thus we see that, when $n > 1$ the characteristic of $\log n$ = (the number of digits in the whole number part of n) - 1. Let us study the examples below.

$$\log 0.0531 = \log (5.31 \times 10^{-2})$$

$$= - 2 + \log 5.31$$

$$\log 0.00032 = \log (3.2 \times 10^{-4})$$

$$= - 4 + \log 3.2$$

Thus we see that -

When $0 < n < 1$, the characteristic part of $\log n$ = (number of zeroes following the decimal point + 1) with a - Ve' sign.

5.8 Some other important concepts

$$\log 735.2 = \log (7.352 \times 10^2)$$

$$= 2 + \log 7.352$$

$$\log 73.52 = \log (7.352 \times 10^1)$$

$$= 1 + \log 7.352$$

$$\log 0.7352 = \log (7.352 \times 10^{-1})$$

$$= - 1 + \log 7.352$$

Thus it can be seen that -

log 735.2, log 73.52, log 7.352 log 0.7352 have the same mantissa. Therefore we come to know -

If digits and their sequence of n remain the same, mantissa remains the same.

Further we come to know -

If the position of the decimal point in n is altered, the characteristic is altered.

Otherwise speaking,

The characteristic of log n depends on the digit contained in 'n' and their sequence, whereas mantissa of log n depends on the position of the decimal point in 'n' .

5.9 <u>Use of logarithm in numerical computation</u>:

We have come across several examples of reverse processes earlier.

Examples are -

Addition and subtraction

Multiplication and division

Power and root

Such reverse processes, when applied on a certain number, we get back the same number. For example -

$5 + 2 - 2 = 5$ [addition of 2 and subtraction of 2]

$7 \times 3 \div 3 = 7$ [multiplying by 3 and dividing by 3]

$\sqrt{15^2} = 15$ [squared and then square root is taken.]

Similarly there is a reverse process of of logarithm which is known as antilogarithm.

Definition:

If $A \in R$, $A > 0$ and $A \neq 1$, then $\log A = B$

$\Leftrightarrow$ antilog $B = A$

$\Rightarrow$ antilog $(\log A) = A$ ....(1)

$\Rightarrow \log$ (antilog B) $= B$ ....(2)

If $A = a \times 10^c$ where $1 < a < 10$ and $c \in Z$

then $\log A = c + \log a$

where c is the characteristic and $\log a$ is the mantissa.

But a = anti log ($\log a$) [$\because$ of (1)]

= antilog (mantissa of log A)

$\therefore$ $A = a \times 10^c$

$= 10^c \times$ antilog (mantissa of log A)

5.10 Use of logarithm and antilogarithm in numerical computation:

Rule (1) above shows that antilog ($\log A$) $= A$

Thus by taking logarithm of A and finding the antilogarithm of the result obtained for $\log A$, we get an approximate rational value for A.

5.11 Some examples of application of logarithm and antilogarithm.

(1) Determine the number of digits in the integral part of $(3.057)^{18}$

Solution: $\log (3.057)^{18} = 18 \times \log 3.057$

$= 18 \times (0.4853) = 8.7354$

The number digits in the integral part of the given number 3.057 = 9

The characteristic of log A is one less than the number of digits in the integral part of A.

(2) If log 23 = 1.3617 and log 5 = 0 .6990 find x if (i) log x = 1.3980 (ii) log x = 2.06 (iii) log x = 0.6627. (No table to be used)

Solution:

(i) $\log x = 1.3980$

$= 2 \times 0.6990$

$= 2\times \log 5$

$= \log 5^2 = \log 25$

$\Rightarrow x = 25$

(ii) $\log x = 2.0607$

$= 1.3617 + 0.6990$

$= \log 23 + \log 5$

$= \log (23 \times 5) = \log 115$

$\Rightarrow x = 115$

(iii) $\log x = 0.6627$

$= 1.3617 - 0.6990$

$= \log 23 - \log 5$

$= \log \frac{23}{5} = \log 4.6$

$\Rightarrow x = 4.6$

(3) (i) $2^x = 64$ (ii) $2^x = 35$

Solution: (i) $2^x = 64 = 2^6$

$\Rightarrow x = 6$

(ii) $2^x = 35$

It can be seen that 35 can not be expressed as an integral power of 2 as was possible for 64. Hence the above process will not help us. Since logarithm helps in removing the power (using the rule $\log x^y = y \times \log x$) we make use of logarithm.

$$2^x = 35$$
$$\Rightarrow \log 2^x = \log 35$$
$$\Rightarrow x \log 2 = \log 35$$
$$\Rightarrow x \times 0.3010 = 1.5441$$
$$\Rightarrow x = \frac{1.5441}{0.3010}$$

Either by actual division or by further using logarithm and antilogarithm the approximate result for x can be determined.

Problems of the following type can also be easily solved by using logarithm and antilogarithm in the above line.

<u>Problem</u>: In how many years the principal of Rs.5000 - 00 will fetch an amount of Rs.20,000-00 at 12% compound interest.

<u>Solution</u>: According to the question above we get -

$$5000 \left(1 + \frac{12}{100} \right)^n = 20,000$$

$$5000 \times (1.12)^n = 20,000$$

$$1.12^n = 4$$

Now the equation with n as the index can be solved in the same line as Q 3 (ii) was solved.

Change of base in logarithm:

We can also change the base of the logarithm of a number in the following manner.

Supposing the base of a log. term is 'a' and we want to change it to 'b', here a and b are positive rational numbers none being equal to 1.

Let the given term be $\log_a m$

Assumed that $\log_a^{m} = x$ .... (1)

$\Rightarrow a^x = m$ .... (2)

Further assumed that $b^y = m$ ....(3)

$\Rightarrow \log_b^{m} = y$ .... (4)

(2) and (3) $\quad a^x = b^y$

$$\Rightarrow (a^x)^{\frac{1}{y}} = (b^y)^{\frac{1}{y}}$$

$$\Rightarrow a^{\frac{x}{y}} = b$$

$$\Rightarrow \frac{x}{y} = \log_a^{b}$$

$$\Leftrightarrow x = y \log_a^{b}$$

Now (1) $\Rightarrow \log_a^{m} = x = y \log_a^{b}$

$\Rightarrow \log_a^{m} = \log_b^{m} \times \log_a^{b}$ of (4)

Thus $\log_a^{m}$ has been expressed in terms of $\log_a^{m}$. Hence the rule used for the purpose is

$$\log_a^{m} = \log_b^{m} \times \log_a^{b}$$

Taking $m = a$ in the rule above we get -

$$\log_a^{a} = \log_b^{a} \times \log_a^{b}$$

$$1 = \log_b^{a} \times \log_a^{b}$$

As we have seen in the proceeding discussions, logarithm and antilogarithm is a useful tool in our hands of simplifying numerical calculations involving multiplication, division, roots and powers. It can also help us in solving exponential equations and so on.

The contribution of Barron John Napier and Henery Briggs are unforgetable for the students of mathematics.

POLYNOMIALS

6.1 Introduction:

1. Let us consider the following examples:-

   $2x$, $3x^2$, 16, 0

   These are called monomials in x as they each occur in single term.More means one.

2. Consider another example.

   $x^3 + 2x^2 + 1$

   This is the summathon of 3 monomials. So this is called polynomial.

3. Again all monomials are also polynomials. Thes 3x is monomial, it can also be written as $ox^2 + 3x$

   Now this is a polynomial.

4. Polynomials having all Coefficients zero is called zero polynomials.

   Example:- $ox^3 - ox + ox^o$ is zero polynomial.

5. Any real number (say 16) is called zero degree polynomial as it can be written as $16\ x^o$. So 0 can be termed as monomial or Zero polynomial or Zero degree polynomial

6. Definition: P(x) of the form $a_o + a_1x + a_2x^2 \dots\dots\dots\dots + a_n\ x^n$ where a_o, a_1, a_2, $\dots\dots\dots$ a_n are rational numbers and n is nonnegative integer is called polynomial in x of n^{th} degree (in rational number set). If a_o, a_1, $a_2 \dots\dots a_n \in R$ then P(x) is a polynomial in Real number set.

7. Let us consider the expression
$x^{-3} + 2x^{-2} + 1$ and $x^{3/2} + 2x^{1/2} + 1$

These two as per definition are not polynomials. These are Algebraic expressions.

8. Monomials of equal degree of same letter, like and similar monomials. Polynomials of equal degree of same letter are like and similar polynomials.

(i) $2x$ and $3x$ (ii) $3x^2$ and $4x^2$ are examples of like monomials . But $1 + 2x + 3x^2$ and $2 + 3x + 4x^2$ are like polynomials.

6.2 Remainder Theorem:

Suppose $P(x) = x^3 + 2x^2 + x - 5$

$q(x) = x - 3$

If we want to find out the remainder by dividing $P(x)$ by $q(x)$ we can see the result in two ways. Firstly, by actual division we can determine the remainder. Secondly the remainder can be determined by the application of remainder theorem. In the second method we need not divide $P(x)$ by $q(x)$. Simply applying formula we can get the result. The Remainder theorem states that if $P(x)$ is a polynomial having its degree ≥ 1, and $P(x)$ is divided by $x-a$, thus the remainder is $P(a)$.

Proof: When $P(x)$ is divided by $q(x) = x - a$ suppose we get quotiest $K(x)$ and remainder $r(x)$, then

$p(x) = K(x)(x-a) + r(x)$

Here the degree of the divisor $(x-a)$ is 1. So the degree of the remainder $r(x)$ is 1 or it may be Zero. So remainder is a constant number. According to the rule of division.

$$P(x) = (x-a) \times q(x) + r(x)$$

Taking $x = a$ in the above equation we get

$$P(a) = (a - a) K(a) + r(a)$$

$$\Rightarrow P(a) = 0 + r(a) = r(a)$$

$$P(x) = (x-a) K(x) + P(a)$$

Hence $P(a)$ is the remainder.

<u>Example follows:</u>

<u>Example</u> Let us find out the remainder of $P(x) = x^3 + 2x^2 + x - 5$ divided by $q(x) = x - 3$.

$$
\begin{array}{r|l|l}
x-3 & x^3 + 2x^2 + x - 5 & x^2 + 5x + 16 \\
 & x^3 - 3x^2 & \\
 & - \quad + & \\
\hline
 & 5x^2 + x & \\
 & 5x^2 - 15x & \\
 & - \quad + & \\
\hline
 & 16x - 5 & \\
 & 16x - 48 & \\
 & - \quad + & \\
\hline
 & 43 &
\end{array}
$$

Here remainder is 43 determine by actual division. Let us apply the remainder theorem for determining the remainder.

$P(x) = x^3 + 2x^2 + x - 5$

$q(x) = x - 3$

Remainder $= P(3) = 3^3 + 2 \times 3^2 + 3-5$

$= 27 + 18 + 3 - 5$

$= 48 - 5 = 43$

6.3 Application of Remainder Theorem in factorisation

To factorise the polynomial $2x^3 + 3x^2 - 2x - 3$

$P(x) = 2x^3 + 3x^2 - 2x - 3$

We try with the value of x as +1, -1, +2, -2 etc. So as to get the value of $P(x) = 0$

By trial we get $P(1) = 3(1)^3 + 3(1)^2 - 2,1-3$

$= 2 + 3 - 2 - 3 = 0$

Hence x-1 is one of the factors of P(x).

Let us divide $2x^3 + 3x^2 - 2x - 3$ by x-1.

Here the quotient is $2x^2 + 5x + 3$

So $2x^3 + 3x^2 - 2x - 3 = (x-1)(2x^2 + 5x + 3)$

Now $2x^2 + 5x + 3$ can be factorised by middle form splitting method.

So we have $2x^3 + 3x^2 - 2x - 3 = (x-1)(2x^2 + 5x + 3)$

$= (x-1)(2x^2 + 2x + 3x + 3)$

$= (x-1)\{2x(x+1) + 3(x+1)\}$

$= (x-1)(x+1)(2x+3)$

Example:

Factorise the polynomial $2x^4 + 4x^3 + 2x^2 + x + 1$

$P(x) = 2x^4 + 4x^3 + 2x^2 + x + 1$

$P(-1) = 3(-1)^4 + 4(-1)^3 + 2(-1)^2 + (-1) + 1$

$= 2 - 4 + 2 - 1 + 1 = 0$

So first factor is $x - (-1) = x + 1$

To get the second factor, let us divide $p(x)$ by $(x + 1)$. The quotient is $2x^3 + 2x^2 + 1$

Hence the factors are $(x + 1)\ (3x^3 + 2x^2 + 1)$

Thus Remainder theorem plays a vital role in factorisation of polynomials having degree 3 or more. For polynomials of degree 2 or less, several methods of factorisation are known to us.

6.4 Certain Properties of Polynomials:

(i) Closure property in addition and multiplication

$p(x) + q(x)$ is a polynomial

$p(x) \times q(x)$ is a polynomial

(ii) Commutative property in addition and multiplication.

$$p(x) + q(x) = q(x) + p(x)$$

$$p(x) \times q(x) = q(x) \times p(x)$$

(iii) Associative property in addition and multiplication.

$$p(x) + \{q(x) + r(x)\} = \{p(x) + q(x)\} + r(x)$$

$$p(x) \times \{q(x) \times r(x)\} = \{p(x) \times q(x)\} \times r(x)$$

(iv) Existence of identity of addition and multiplication.

$p(x) + 0 = 0 + p(x) = p(x)$

0 is the identity of addition

$p(x) \times 1 = 1 \times p(x) = p(x)$

1 is the identity of multiplication.

(v) Existence of additive inverse

$p(x) + \{-p(x)\} = 0$

$\therefore$ $p(x)$ and $-p(x)$ are additive inverse of each other.

(Multiplicative inverse does not exist)

(vi) Distributive property of multiplication over addition.

$p(x)\{q(x) + r(x)\} = p(x) \times q(x) + P(x) \times r(x)$

A rule regarding the degree of the product of two polynomials.

Degree of $p(x) \times q(x)$ = degree of $p(x)$ + degree of $q(x)$

6.5 H C F of Polynomials

HCF of monomials:

Let us find out HCF of two monomials

$24\,x^2y\ ,\ 36\,x^2\,y^2\,z$

HCF of 24, 36 = 12

HCF of $x^2\,y\ ,\ x^2\,y^2\,z = x^2\,y$

Hence required HCF = $12\,x^2\,y$

Here in two monomials x, y are the common algebraic symbols. x^2 and y are at lowest degree commonly available in both the monomials.

So HCF is $x^2\,y$

For determining the HCF of polynomials the following steps are followed.

1st step: Each polynomial is expressed in factors.

2nd step: The highest degree of each factor that occurs commonly in all the polynomials is noted.

<u>3rd step:</u> HCF is determined by taking the product of maximum number of times of each factor that commonly occurs in them.

<u>For example:</u>

$$x^2 - 2ax + a^2 = (x-a)^2$$

$$x^2 - 3ax + 2a^2 = (x-a)\ (x-2a)$$

$$x^3 - 3ax^2 + 3a^2x - a^3 = (x - a)^3$$

Maximum occurance of $x-a$ commonly in all the polynomials = $(x-a)$ (i.e. once only) $(x-2a)$ does not occur commonly in all the polynomials. So is not taken.

Hence HCF = $x - a$.

6.6 <u>An Important fact:</u>

HCF of $(x-2)^2$ and $(2-x)\ (1-x)$ = ?

Apparently no common factor is found existing in the two polynomials. But we can write the first polynomial as shown below:

1st polynomial = $(x-2)^2 = (2-x)^2$

$\because (a-b)^2 = (b-a)^2$.

Now we find $(2-x)$ as the HCF of the two polynomials. or the 2nd polynomial = $(2-x)\ (1-x)$

$= \{-(x-2)\}\ \{-(x-1)\}$

$= (x-2)\ (x-1)$

Now we see that the HCF is $x - 2$. Hence, at the given polynomials the HCF is either $x - 2$ or $2 - x$.

6.7 L.C.M. of Monomials

L.C.M. as we know, is the lowest common multiple.

Let us take the following examples:

(i) LCM of x^2, x^3, x^5 = lowest common multiple of x^2, x^3, x^5 = x^5 i.e. the highest power of x.

(ii) L.C.M. of monomials $x^2 y$, xy, $x^3 y^2$

$= (\text{LCM of } x^2, x, x^3) \times (\text{LCM of } y, y, y^2)$

$= x^3 \times y^2 = x^3 y^2$

(iii) L.C.M. of monomials having numerical coefficients e.g. $3x^2y$, $6xy^3$, $15\ y^2z$, $20\ x\ z^3$

$$\text{L.C.M.} = \{\text{LCM of } 3,6,15,20\} \times \{\text{LCM of } x^2, x, x\} \times \{\text{LCM of } y, y^3, y^2\} \times \{\text{LCM of } z, z^3\}$$

$$= 60 \times x^2 \times y^3 \times z^3 = 60\ x^2\ y^3\ z^3$$

For determining the L.C.M. of poly nomials, We follow the procedure similar to what has been discussed in respect of monomials.

Steps followed .

1st step: Expressing each polynomial in factors.

2nd step: The maximum number of occurance of each factor is the polynomials is noted.

3rd step: The LCM is determined by taking the product of the maximum number of occurance in the polynomials.

For example -

$$x^2 - 3x + 2 = (x - 1)(x - 2)$$

$$x^2 - 4x + 4 = (x - 2)^2$$

$$x^3 - 3x^2 + 3x - 1 = (x - 1)^3$$

Maximum occurance of the factor $x - 1$ is $(x-1)^3$

Maximum occurance of the factor $x - 2$ is $(x-2)^2$

$\therefore$ The L.C.M. $= (x - 1)^3 (x - 2)^2$

....

6.8 Rational expressions

In the analogy that $\frac{a}{b}$ becomes a rational number when a, b, $\in$ z, and $b \neq 0$, $\frac{p(x)}{q(x)}$ is an algebraic rational expression when $q(x) \neq 0$.

For example -

$\frac{x^2 + 2x + 3}{x + 2}$ is as algebraic rational expression.

When $x + 2 \neq 0$ i.e. $x \neq -2$

Since 1 is also a polynomial, p(x) which is the same as $\frac{p(x)}{1}$ takes the form of $\frac{p(x)}{q(x)}$

$\therefore$ p(x) is also a rational expression. That is a polynomial is also an algebraic rational expression. This bears an analogy with the fact that 5 which can also be written as $\frac{5}{1}$ is also a rational number. Thus any integer is also a rational number.

6.9 Lowest form of a algebraic rational expression

We reduce the rational number $\frac{12}{20}$ to the lowest form by expressing the numerator and the denominator into factors in the manner as shown below.

$$\frac{12}{20} = \frac{2 \times 2 \times 3}{2 \times 2 \times 5} = \frac{3}{5}$$

(Common factors are cancelled)

Similarly by expressing p(x) and q(x) in factors the algebraic rational expression $\frac{p(x)}{q(x)}$ (where $q(x) \neq 0$) can be reduced to its lowest form.

Example follows:

$$\frac{x^2 + 5x + 6}{x^2 + 6x + 8} = \frac{(x+2)\ (x+3)}{(x+4)\ (x+2)} \quad \left[\text{when } x \neq -4 \text{ and } x \neq -2\right]$$

$$= \frac{x+3}{x+4} \quad \left[\text{Cancelling out the non-zero common factor}\right]$$

<u>Note:</u> Whenever we come across a rational expression we reduce it to the lowest form.

<u>Addition of rational expression:</u>

Addition of rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ was defined as follows:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

Addition of rational expression also follow the same line. Thus

$$\frac{p(x)}{q(x)} + \frac{r(x)}{s(x)} = \frac{p(x) \times s(x) + r(x) \times q(x)}{q(x) \times s(x)}$$

Example follows:

$$\frac{x^2+1}{x+1} + \frac{x-2}{x-1} = \frac{(x^2+1)\ (x-1) + (x-2)(x+1)}{(x+1)\ (x-1)}$$

$$= \frac{x^3 - x^2 + x - 1 + x^2 - 2x + x - 2}{(x+1)\ (x-1)}$$

$$= \frac{x^3 - 3}{(x+1)\ (x-1)}$$

If possible the result obtained is to be reduced to the lowest term.

6.10 Multiplication of two rational expressions:

We have defined the product of two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ as followw:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

In the same line the product of rational expressions is also defined such as -

$$\frac{p(x)}{q(x)} \times \frac{r(x)}{s(x)} = \frac{p(x) \times r(x)}{q(x) \times s(x)}$$

Division of rational expressions also follow the same line as division of two rational numbers. Such as :

$$\frac{p(x)}{q(x)} \div \frac{r(x)}{s(x)} = \frac{p(x)}{q(x)} \times \frac{s(x)}{r(x)}$$

Thus a great deal of analogy is found existing in the operations occuring in rational numbers and operations occuring in rational expressions.

6.11 Arithmatic and Geometric progressions

Sequence: A Series of numbers which occur in such an order that after seeing a few of them we can say what would follow, is known as a sequence.

There are certain examples below:

(i) 1, 2, 3, 4 ....

(ii) 2, 4, 6, 8 ....

(iii) 1, 3, 9, 27 ....

(iv) 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...

Each of the above is a sequence. Though in each of serieses only four terms are available enough of indications are available tosay what are the numbers that would follow.

Hence each of them is a sequence.

But structures of them, are different from one another. In example (i) each terms is greater then the previous one by 1. Thus the

2nd one = 1st + 1

3rd one = 2nd + 1 = 1st + 1 + 1 = 1st + 2

4th one = 3rd + 1 = 1st + 2 + 1 = 1st + 3

and so on.

In example it can be seen that

2nd one = 1st + 2

3rd one = 1st + 2 x 2

4th one = 1st + 3 x 2

and so on.

Such sequences are known as Arithmatic progression.

Thus denoting the 1st term as 'a' and the common differences as 2, We can say that the n th term = a + (n - 1) d

Where 'a' is the 1st term and 'd' is the common difference.

6.12. Sum of an A.P.

Let us take the following A.P. having 'n' no. of terms .

$S = a + (a+d) + (a+2d) + (a+3d) + \ldots + (l-d) + l$

$S = l + (l-d) + (l-2d) + \ldots\ldots (a+d) + a$

Adding [written to the reverse order.]

$2S = (a+l) + (a+d+l-d) + (a+2d+l-2d) + \ldots\ldots$

$(l-d+a+d) + (d+a)$

$2S = (a+l) \times n$

$\Rightarrow S = \frac{n}{2}(a+l)$ or $S = \frac{n}{2}\{2a+(n-1)d\}$ where $l = a+(n-1)d$.

Where n is the number of terms and 'l' is the last term.

In example (iii) we see that

$\frac{\text{2nd term}}{\text{last term}} = \frac{\text{3rd term}}{\text{2nd term}} = \frac{\text{4th term}}{\text{3rd term}} = \ldots\ldots$

$= 3$ which is a constant .

Thus, 2nd term = 1st term X 3

3rd term = 2nd term X 3 = 1st term X 3^2

4th term = 3rd term X 3 = 1st term X 3^3

Denoting the 1st term as 'a' and common ratio as 'r', we get n th term = $a\,r^{n-1}$.

Such a series of terms is known as a Geometric Progression. A. G.P. is expressed in symbols as -

$a, ar, ar^2, ar^3 \ldots\ldots\ldots ar^{n-1}$

Sum of a G.P.

Let $S = a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1}$

$(-)rS = \underline{-}ar \pm ar^2 \pm ar^3 \pm ar^4 \ldots\ldots \pm ar^{n-1} \pm ar^n$

$S(1-r) = a \ldots\ldots\ldots\ldots - ar^n$

$$\Rightarrow S(1-r) = a(1-r^n)$$

$$\Rightarrow S = \frac{a(1-r^n)}{1-r} \quad \text{when} \quad r < 1$$

$$= \frac{a(r^n-1)}{r-1} \quad \text{when} \quad r > 1$$

Some specific series in A.P. are illustrated below.

(i) 1, 2, 3, 4, 5 . . . .

This, is the Natural number series. If we take the number of terms as 'n', we get

$$S = \frac{n}{2}(a + l)$$

$$\Rightarrow S = \frac{n}{2}(1 + n) \quad [\because \text{Last term } l = n \text{ when no. of terms is } n.]$$

$$S = \frac{n(n+1)}{2}$$

Thus the sum of $1 + 2 + 3 + \ldots\ldots + 40 = \frac{40 \times 41}{2}$

$$= 20 \times 41 = 820$$

(ii) For finding sum of $20 + 21 + 22 + \ldots\ldots + 60$

We may follow the following procedure.

Let $S_2 = 1 + 2 + 3 + \ldots\ldots + 19$

$$S_2 = \frac{19 \times 20}{2} = 19 \times 10 = 190$$

The required sum $= S_1 - S_2 = 1830 - 190$

$= 1640$ Ans.

(iii) For fixing the sum of all two digit numbers which are multiples of 5, we may follow the procedure illustrated below.

The required sum $= 10 + 15 + 20 + \ldots\ldots + 95$

$= 5(2 + 3 + 4 + \ldots\ldots\ldots + 19)$

$= 5\{(1 + 2 + 3 + \ldots\ldots\ldots + 19) - 1\}$

$= 5\left\{\frac{19 \times 20}{2} - 1\right\}$

$= 5 \times (190 - 1) = 5 \times 189 = 945$

6.13 Sum of an infinite G.P.

If we take the G.P.

a, ar, ar^2, ar^3 .... infinitely

There are two possibilities.

(1) when $r > 1$ and $n \to \infty$

$r^n \to \infty$

$\therefore S = \frac{a(r^n - 1)}{r - 1}$ becomes infinitely large

(2) When $r < 1$ and $n \to \infty$

$r^n \to 0$ [a fraction raised to higher powers becomes smaller in value.]

$$S = \frac{a(1 - r^n)}{1 - r} = \frac{a \times 1}{1 - r} = \frac{a}{1-r}$$

For example -

$1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots$ to infinite no. of terms

$$= \frac{a}{1-r} = \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 1 \times \frac{2}{1} = 2 \text{ Ans.}$$

7. Linear Equation in One Variable

An equation is a statement showing the equality between two expressions each of which or one of which contains one or more variable. We shall limit our discussions here only to equations involving one variable.

An equation of the type

$$ax + b = 0$$

is called a linear equation or first degree equation as the highest degree of the unknown 'x' contained in the equation is 1. Thus each of the following is a linear equation.

$$3x - 5 = 0 \quad \ldots\ldots\ldots \quad (1)$$

$$\frac{x}{2} + \frac{x}{3} = 5 \quad \ldots\ldots\ldots \quad (2)$$

$$2(\,x - 3\,) = x - 4 \quad \ldots (3)$$

It may be noted that $ax + b = c$ a linear equation where $a \neq 0$.

It can further be seen that each of the 3 equations given above is satisfied by a unique value of x e.g.

eq. (1) is satisfied only by 5/3

eq. (2) is satisfied only by 6

and eq. (3) is satisfied only by 2.

Let us examine the following

$$2(x-1) + 1 = 3 - (4 - 2x) \quad \ldots\ldots (4)$$

It can be seen that any value taken for x satisfies (4). Hence the equality statement in (4) is not an equation. It is known as an identity.

Solution of a linear equation

Let us consider the linear equation below.

$$5x + 7 = 12$$

If we try with $x = 2$, we get LHS = 17 and RHS = 12, Hence LHS $\neq$ RHS.

If we substitute 1 as the value of x then LHS becomes equal to RHS. The particular value of x i.e. 1 which makes both sides of the equation equal is called the solution (or root) of the equation. Hence '1' is the solution for the above equation.

Let us consider another example $2x - 7 = x + 1$. How we find its solution ?

$$2x - 7 = x + 1$$
$$\Rightarrow 2x - x = 1 + 7$$
$$\Rightarrow x = 8$$

Taking $x = 8$, in the LHS and RHS, we note that LHS = RHS. Hence $x = 8$ is the solution.

7.1 Application of linear equation:

Certain arithmetical problem can be expressed as equations by taking the unknown as x. It becomes more convenient to solve the equation and determine the unknown than solving it by arithmatical process. The following example illustrates the fact.

Example:- 1 At what rate % of S.I. per year the interest on Rs.500.00 for 3 years exceeds the interest on Rs.600.00 for 2 years by Rs.24.00 ?

Solution (arithmatical process)

The interest on Rs.500.00 for 3 years is the same as the interest on Rs.100.00 for 3×5 yrs. = 15 yrs.

The interest on Rs.600.00 in 2 years is the same as the interest on Rs.100.00 for 2×6 yrs. = 12 yrs.

The difference between the interest on Rs.500.00 for 3 yrs. and that on Rs.600.00 for 2 yrs. = the difference between the interest on Rs.100.00 for 15 years and the interest on Rs.100.00 in 12 yrs. = the interest on Rs.100.00 for (15 - 12) yrs. i.e. 3 yrs.

But the difference between the interests = Rs. 24.00

$\therefore$ The interest on Rs.100.00 in 3 yrs. is Rs.24.00. Hence the interest on Rs.100.00 in 1 yr. = Rs.24.00 $\div$ 3 = Rs. 8.00

$\therefore$ the rate of S.I. per yr. = 8 %

Solution (by application of equation)

Let the rate of simple interest be r%

$\therefore$ S. i on Rs.500.00 for 3 yrs. = $\frac{500 \times 3 \times r}{100}$

= 15 r Rs.

and S.i. on Rs.600.00 for 2 yrs. = $\frac{600 \times 2 \times r}{100}$

= 12 r Rs.

But the difference of interests = Rs.24.00

$\therefore$ $15r - 12r = 24$

$\Rightarrow$ $3r = 24$

$\Rightarrow$ $r = 8$

$\therefore$ rate of Si = 8% per annum (Ans.)

Example:- 2

Determine 3 consecutive multiplies of 7 whose sum is 399.

Solution (arithmatical process)

Since the three numbers are the consecutive multiplies of 7, the middle one is greater than the smallest by 7 and the greatest one is greater than the smallest by 14. Now subtracting 7 from the middle one and 14 from the greatest , each of the two resulting numbers is equal to the smallest and their sum is decreased by $7 + 14 = 21$

$\therefore$ the sum becomes $399 - 21 = 378$

Each of the 3 equal numbers $= 378 \div 3 = 126$

Hence the smallest = 126

The middle one $= 126 + 7 = 133$

The greatest $= 126 + 14 = 140$

<u>Solution</u> (by application of equation)

Let the smallest of the 3 numbers be x.

the middle one $= x + 7$ and the greatest one $x + 7 + 7 = x + 14$. The sum of the 3 numbers = 399

$\Rightarrow x + x + 7 + x + 14 = 399$

$\Rightarrow 3x + 21 = 399$

$\Rightarrow 3x = 399 - 21$

$\Rightarrow 3x = 378$

$\Rightarrow x = 126$

The middle one $= x + 7 = 126 + 7 = 133$

The greatest one $= x + 14 = 126 + 14 = 140$

Thus it can be realised that the algebraic process (using equation) is easier than the arithmatical process. The first process involves more intricate thinking process and analysis of the situation. Where as the second process (using equation involves more direct thinking process and hence easier to solve.

Example:- 3 The sum of two numbers is 80 and their difference is 40. Find the numbers.

Solution :- Let one number be x, then the other number is (80 - x), According to questions

$$x - (80 - x) = 40$$

$$\Rightarrow x - 80 + x = 40$$

$$\Rightarrow 2x - 80 = 40$$

$$\Rightarrow 2x = 40 + 80$$

$$\Rightarrow 2x = 120$$

$$\Rightarrow x = \frac{120}{2}$$

$$\Rightarrow x = 60$$

So one number is 60 and the other one is

80 - 60 = 20

Example:- 4 The measure of the three angles of a triangle is in the ratio 1:2:3. Find out the measure of the angles.

Solution:- In this case again we may use the unknown quantity i.e. x. According to the question the three angles are x, 2x and 3x.

Hence $x + 2x + 3x = 180$

$\Rightarrow 6x = 180$

$\Rightarrow x = \frac{180}{6}$

$\Rightarrow x = 30$

So the measure of angles are 30°, 60° and 90°

Example:- 5

Rama's age is now 1/3 of his father's age 10 years ago the age of Rama's father was 5 times that of Rama's age. Calculate the present age of Rama and his father.

Solution:- Let the present age of Rama's father be x years. So according to the question the age of Rama is $\frac{x}{3}$ years. Again 10 years ago the age of Rama's father was $(x - 10)$ years and the age of Rama was $(\frac{x}{3} - 10)$ years.

According to question:

$$x - 10 = 5 \left(\frac{x}{3} - 10 \right)$$

$$\Rightarrow x - 10 = \frac{5x}{3} - 50$$

$$\Rightarrow x - 10 = \frac{5x - 150}{3}$$

$$\Rightarrow 3(x - 10) = 5x - 150$$

$$\Rightarrow 3x - 30 = 5x - 150$$

$$\Rightarrow 150 - 30 = 5x - 3x$$

$$\Rightarrow 120 = 2x$$

$$\Rightarrow 2x = 120$$

$$\Rightarrow x = \frac{120}{2}$$

$$\Rightarrow x = 60$$

Hence the present age of Rama's father is 60 yrs. and that of Rama is 20 years.

Alternative Solution:-

Let Rama's present age be x. Then his father's present age is 3x.

10 years ago -

Rama's age was (x - 10) years.

His father's age was (3x - 10) years.

But father's age was 5 times the age of Rama.

$3x - 10 = 5(x - 10)$

$\Rightarrow 3x - 10 = 5x - 50$

$\Rightarrow 50 - 10 = 5x - 3x$

$\Rightarrow 40 = 2x$

$\Rightarrow 2x = 40$

$\Rightarrow x = \frac{40}{2}$

$\Rightarrow x = 20$

$\therefore$ Rama's present age is 20 and his father's present age is $20 \times 3 = 60$ (Ans.)

Note:- The choice of x for a suitable unknown makes the work simpler.

7.2 Linear equations involving two unknowns:

Let us study the equation given below.

$$2x - 3y = 7$$

It contains two unknowns 'x' and 'y' and each of them has a power 1 at the highest. Hence it is a linear equation in two unknowns. Can we solve for x or y making use of the given equation ?

We can express x in terms of 'y' if 'x' is made the subject of the equation.

Thus we get $x = \frac{3y + 7}{2}$

which is an expression in terms of y .

Similarly 'y' can be expressed in terms of x.

Thus we get

$$y = \frac{2x - 7}{3}$$

As is already seen the value of an unknown can be obtained if we have a linear equation in the same unknown only. Thus to get the value of x we need a linear equation containing x only. For this we need the elimination of y from the given equation.

For elimination of y (or x) we need a second equation containing y. Let us take the equation earlier given along with a second one.

$$2x - 3y = 7 \quad \ldots\ldots.(1)$$

$$3x + y = 5 \quad \ldots\ldots.(2)$$

From equation (1) we have $x = \frac{3y + 7}{2}$

Taking various values for y corrresponding values of x can be determined. Some are shown below:

if $y = 9$, $x = 17$

$y = 7$, $x = 14$

$y = 4$, $x = 9.5$

$y = 0$, $x = 3.5$

$y = -3$, $x = -1$

Thus ordered pairs (17,9), (14,7), (9.5, 4), (3.5, 0), (-1, -3) and infinitely many ordered pairs shall be made available in the solution set of the given equation.

Similarly, from eqn.(2) we have

$$x = \frac{5 - y}{3}$$

Taking various values of y corresponding values of x can be made available. Some are illustrated below.

if $y = 10$, $x = -5/3$

$y = 9$, $x = -4/3$

$y = 8$, $x = -1$

$y = 5$, $x = 0$

$y = 2$, $x = 1$

$y = -1$, $x = 2$

Thus ordered pairs $(-\frac{5}{3}, 10)$, $(-\frac{4}{3}, 9)$, $(-1,8)$, $(0,5)$, $(1,2)$, $(2, -1)$ and infinitely many ordered pairs shall be made available in the solution set of eqn.(2).

It can be seen that in both the solution sets only one ordered pair would be obtained as the common element. Why it is so ?

A geometrical answer can be given for the above question.

If we plot all ordered pairs obtained in the solution set of Eqn.(1) on a graph paper they will be seen lying in one straight line. This can also be proved logically.

Thus a linear equation with two unknowns x and y represents a straight line.

Thus the two equations (1) and (2) represent two straight lines. Two lines have the following relative structures.

(a) They may be coincident

(b) They may be parallel

(c) They may cut each other at one point only.

A pair of equation can only have one of the above three relative structures.

Examples of pairs of equations that represent the above structure are shown below.

(A) $2x - 3y = 7$

and $6x - 9y = 21$

It can be seen that the ordered pair (-1, -3) satisfies both the equations and any other ordered pair which satisfies one of them also satisfies the other.

The two equatons shown in (A) represent coincident lines.

(Such pair of equations are consistent and dependent).

(B) $2x - 3y = 7$

$4x - 6y = 9$

It can be seen that no ordered pair will be available which satisfies both. It so happens because the lines represented by the two equations are parallel and as such they have no common point. This can also be proved logically.

(Such pair of equations are in consistent).

(C) $2x - 3y = 7$ ....(1)

and $3x + y = 5$ ....(2)

The two lines available from the two equations given above will neither be coincident nor be parallel. Those will be intersecting straight lines. As such there will be only one point common to them.

(Such pair of equations are consistent and independent).

Such pairs of equations containing two unknowns are known as simultaneous equations.

Here is the answer available for the question raised earlier.

Now let us see to the process of elimination of one unknown from a pair of simultaneous equations given in (C) above.

1st Process:- Eq.(1) $2x - 3y = 7$

$$\Rightarrow x = \frac{3y + 7}{2} \quad \ldots\ldots(3)$$

Eq.(2) $3x + y = 5$

$$\Rightarrow x = \frac{5 - y}{3} \quad \ldots\ldots(4)$$

From (3) and (4) we have

$$\frac{3y + 7}{2} = \frac{5 - y}{3} \quad \ldots\ldots(5)$$

Thus we get equation (5) which contains only y.

2nd Process:- Eq.(1) $x = \frac{3y + 7}{2}$

Substituting in eq.(2) we get

$$3\left(\frac{3y + 7}{2}\right) + y = 5$$

Which is an equation in y only. Thus x is eliminated.

3rd Process:- Eq.(1) × 3 ⇒ $6x - 9y = 21$

Eq.(2) × 2 ⇒ $6x + 2y = 10$

$$\begin{array}{r} 6x - 9y = 21 \\ 6x + 2y = 10 \\ - \quad - \quad - \\ \hline -\quad 11y = 11 \end{array}$$

Thus we get an equation containing y only.

After solving for the unknown in which the equation was available after elimination of the other. We substitute the value in one of the given equations and solve for the other unknown.

7.3 Graphical Solution:

By drawing the graphs of the two given equation we can also find the solution.

(i) If the equations are consistent and dependent , we get two coincident lines. So such pair of equations have infinitely many solutions.

(ii) If the equations are consistent and independent, we get intersecting lines. So such pair of equation give a unique solution.

(iii) If the equations are inconsistent then we get parallel lines and such pair of equations give no solution.

.....

QUADRATIC EQUATION

8.1 Introduction:

$ax^2 + bx + c$ is a polynomial of second degree where a, b, c are real numbers and $a \neq 0$. Here 'x' is a variable. Such a polynomial p(x) is called a quadratic polynomial $p(x) = 0$ i.e. $ax^2 + bx + c = 0$ is a quadratic equation where 'a' is the co-efficient of x^2, 'b' is the coefficient of x and 'c' is a constant term.

Examples: $\sqrt{3}x^2 - 3x - 10 = 0$, $4x^2 + 5x + 6 = 0$, $x^2 + 5x = 0$ are the quadratic equations.

8.2 Zeros of a quadratic polynomial

Suppose $p(x) = 2x^2 - x - 3$ is a polynomial. If we substitute $x = -1$ or $x = 3/2$ the polynomial reduces to zero. These two values of 'x' are called the Zeros of the quadratic polynomial $p(x) = 2x^2 - x - 3$.

8.3 Solution of quadratic equations

Solving a quadratic equation by factorisation

To solve the quadratic equation by factorisation the following principle is utilised.

If $a, b, c \in R$ then $ab = 0 \Rightarrow a = 0$ or $b = 0$

Suppose $ax^2 + bx + c = (Ex + F)(Gx + H)$

Where $E \neq 0$, $G \neq 0$, so $ax^2 + bx + c = 0$

$\Rightarrow (Ex + F)(Gx + H) = 0$

$\Rightarrow Ex + F = 0$ or $Gx + H = 0 \Rightarrow x = -\frac{F}{E}$ or

$x = -\frac{H}{G}$, $\frac{-F}{E}$ and $\frac{-H}{G}$ are called the roots of the quadratic.

Equation $ax^2 + bx + c = 0$. Applying this method we can solve a quadratic equation.

Example:- 1 The quadratic equation $x^2 - 3x - 18 = 0$ can be solved by factorisation.

Solution: $x^2 - 3x - 18 = 0 \Rightarrow (x + 3)(x-6) = 0$

$\Rightarrow x + 3 = 0$ or $x - 6 = 0$

$\Rightarrow x = -3$ or $x = 6$

Therefore the two roots are -3 and 6 .

Example:- 2 Find the roots of the quadratic equation $3x^2 - 8x + 5 = 0$

Solution:- The quadratic polynomial $3x^2 - 8x + 5$ can be factorised as $(3x - 5)(x-1)$.

$\therefore 3x^2 - 8x + 5 = 0 \Rightarrow (3x - 5)(x-1) = 0$

$\Rightarrow 3x - 5 = 0$ or $x - 1 = 0$

$\Rightarrow x = \frac{5}{3}$ or $x = 1$

Hence the two roots of the quadratic equation $3x^2 - 8x + 5 = 0$ are 5/3 and 1.

8.4 By the method of completion of Squares

It is not possible to solve all the quadratic equations using the method of factorization. In subh a case the method of completion of squares is applied.

Examples:- 3 $x^2 + 5x + 5 = 0$ is a quadratic equation which is not easy to solve by factorisation method.

($ax^2 + bx + c$ can be expressed as $(Ex + F)(Gx + H)$ only if $E, F, G, H \in Q$ by the mid-term splitting method.) Hence the method of completion of square is applied.

Method of completion of squares

1. Shridhara Charya's Method or Hinder's method:-

$$ax^2 + bx + c = 0$$

$$\Rightarrow ax^2 + bx = -c$$

$$\Rightarrow 4a^2x^2 + 4abx = -4ac \text{ (multiplying both sides by } 4a\text{)}$$

$$\Rightarrow (2ax)^2 + 2 \times 2ax \times b + b^2 = b^2 - 4ac$$

(Adding b^2 on both the sides)

$$\Rightarrow (2ax + b)^2 = b^2 - 4ac$$

$$\Rightarrow 2ax + b = \pm\sqrt{b^2 - 4ac}$$

$$\Rightarrow 2ax = -b \pm \sqrt{b^2 - 4ac}$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hence the two roots are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

Denoting the two roots as L and B, we get

$$L = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and $B = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

8.3 The other Method

$$ax^2 + bx + c = 0$$

$$\Rightarrow x^2 + \frac{b}{a} + \frac{c}{a} = 0 \text{ (Dividing by 'a')}$$

$$\Rightarrow x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + 2x \times \frac{b}{2a} + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

(Adding $\left(\frac{b}{2a}\right)^2$ on both sides).

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus $L = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and

$$B = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

.5 Sum and Product of the roots

If L and B are the roots of the quadratic equation $ax^2 + bx + c = 0$

Then $L = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $B = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

(We may take $L = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ and

$B = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ also)

The sum of the roots is $L + B = -\dfrac{b}{a}$

$$= - \frac{\text{The Co-efficient of 'x'}}{\text{The Co-efficient of '}x^2\text{'}}$$

The product of the roots $LB = \dfrac{c}{a}$

$$= \frac{\text{The Constant term}}{\text{The Co-efficient of '}x^2\text{'}}$$

Key Points (1) $b = 0 \Rightarrow \dfrac{-b}{a} = 0 \Rightarrow L + B = 0$

Then the roots of the quadratic equation are additive inverse of each other. Conversely if the roots are additive inverse then

$L + B = 0 \Rightarrow \dfrac{-b}{a} = 0$

$\Rightarrow -b = 0 \Rightarrow b = 0$

... roots are multiplicative inverse of each other i.e. receprocal of each other, then $L B = 1 \Rightarrow \frac{c}{a} = 1$

$\Rightarrow c = a$. Conversely if $c = a$ then

$$L B = \frac{c}{a} = \frac{a}{a} = 1$$

That is L and B are reciprocals of each other.

<u>Application:-</u> Find out the sum and product of the roots of the quadratic equation.

$$3x^2 - 7x - 5 = 0$$

Here $a = 3$, $b = -7$ and $c = -5$

Hence the sum $= \frac{-b}{a} = \frac{7}{3}$

The product of the roots $= \frac{c}{a} = \frac{-5}{3}$

8.7 <u>To construct a quadratic equation when the roots L and B are known.</u>

Since the roots of a quadratic equation are L and B

$x = L \Rightarrow x - L = 0$

and $x = B \Rightarrow x - B = 0$

Hence the quadratic equation is

$$(x - L) (x - B) = 0 \quad ....(1)$$

Eq.(1) $\Rightarrow x^2 - L x - Bx + L B = 0$

$$\Rightarrow x^2 - (L + B) x + L B = 0 \quad .....(2)$$

Either of the two forms of equations (1) or (2) can be taken to form a quadratic equation when the roots are given.

Eq.(2) helps us forming the equation when the sum and the product of the roots are given.

Examples follow.

Example:- 3   Construct the quadratic equation whose roots are 3 and -5.

Solution: The required quadratic equation is

$$(x - L) (x - B) = 0 \quad (\text{Eq.}(1) \text{ above})$$

$$\Rightarrow (x - 3) \{ x - (-5) \} = 0$$

$$\Rightarrow (x - 3) (x + 5) = 0$$

$$\Rightarrow x^2 - 3x + 5x - 15 = 0$$

$$\Rightarrow x^2 + 2x - 15 = 0$$

Alternative solution:

The required equation is

$$x^2 - (L + B) x + L B = 0 \quad (\text{Eq.}(2) \text{ above})$$

$$x^2 - \{3 + (-5)\} x + (3)(-5) = 0$$

$$x^2 - (-2) x - 15 = 0$$

$$x^2 + 2x - 15 = 0$$

Example:- 4   Construct the quadratic equation if the sum and product of its roots are 4 and - 6 respectively.

Solution: The required equation is

$$x^2 - (L + B) x + L B = 0 \quad (\text{Eq.}(2) \text{ above})$$

$$\Rightarrow x^2 - \{4 + (-6)\} x + 4(-6) = 0$$

$$\Rightarrow x^2 - (-2) x - 24 = 0$$

$$\Rightarrow x^2 + 2x - 24 = 0 \qquad (\text{Ans.})$$

Eq.(2) may also be written in the form

x^2 - (sum of the roots) x + product of the roots

Example:- 5 Construct the quadratic equation of which the roots are $3+\sqrt{7}$ and $3-\sqrt{7}$.

Solution: (The work becomes more simple if the sum of and the product of the roots are determined first.)

Sum of the roots $= 3+\sqrt{7}+3-\sqrt{7} = 6$

Product of the roots $= (3+\sqrt{7})\,(3-\sqrt{7})$

$= 3^2 - (\sqrt{7})^2 = 9 - 7$

$= 2$

Thus the required equation is

x^2 - (Sum of the roots) x + product of the roots = 0

$x^2 - 6x + 2 = 0$

8.8 Nature of the roots of the quadratic equation

Suppose L and B are the two roots of the quadratic equation

$$ax^2 + bx + c = 0$$

Then $L = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $B = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$

The nature of the roots always depends upon $b^2 - 4ac$ which is called the discriminant of the equation and is denoted by D $[\because a,b,c \in R]$

(i) If $D > 0$ then $\sqrt{D}$ is a real number which is positive. So the roots of the quadratic equation are real and unequal. If a, b, c are rational numbers and D is a perfect square the roots are rational. If a, b, c are rational numbers and D is not a square, the roots are irrational.

(ii) If D = 0 the roots are real and equal.
Here L = B and the root $-\frac{b}{2a}$ is called the double root.

(iii) If D $<$ 0 the roots are not real because the square root of a negative number is not a real number. So the roots are complex.

<u>Example</u> The roots of $x^2 + x + 2 = 0$ and $x^2 - x + 1 = 0$ are complex number and unequal, because in case of the 1st equation $D = b^2 - 4\,ac = -7$ and in case of the 2nd equation $D = b^2 - 4\,ac = -3$. Thus we find that -

(i) If D $>$ 0, the roots are real and unequal
(ii) If D = 0, the roots are real and equal
(iii) If D $<$ 0, the roots are imaginary and unequal.

8.9 <u>Factorization of quadratic polynomials:</u>

If L and B are the roots of the quadratic equation $ax^2 + bx + c = 0$ then $L + B = -\frac{b}{a}$ and $L\ B = \frac{c}{a}$. So the polynomial $ax^2 + bx + c =$
$a\left(x^2 + \frac{b}{a}x + \frac{c}{a} \right) = a\left\{ x^2 - (L + B)\,x + L\,B \right\}$
$= a\,(x - L)\,(x - B)$. Here $(x - L)$ and $(x - B)$ are the factors of the quadratic polynomial $ax^2 + bx + c$.

<u>Example:</u> Factorise the polynomial $3x^2 + 5x + 2$

<u>Solution:</u> Here the roots are

$$L = \frac{-b + \sqrt{b^2 - 4\,ac}}{2a} = \frac{-5 + \sqrt{25 - 24}}{6}$$

$$= \frac{-5 + 1}{6} = -\frac{2}{3}$$

and $$B = \frac{-5 - 1}{6} = -1$$

Therefore the factorisation is as follows:

$3x^2 + 5x + 2 = 0 \Rightarrow 3 (x - L) (x - 3)$

$= 3 (x + \frac{2}{3}) (x + 1) = (3x + 2) (x + 1)$

Here ($3x + 2$) and ($x + 1$) are the factors of the given polynomial.

8.10 Equations reducible to quadratic equations

Sometimes we have to solve equations which are not quadratic equations in nature but which can be reduced to quadratic equations by suitable transformation. Such equations are called equations reducible to quadratic equations. These are different types of such equations.

<u>Example</u>: (Type - I)

$$2 z^4 - 5 z^2 + 3 = 0$$

It is not a quadratic equation . We make the substitution $x = z^2$. Then the given equation reduces to $2x^2 - 5x + 3 = 0$ which is a quadratic equation.

<u>Example</u>: (Type - II)

$3y + \frac{4}{y} = 5$ can also be reduced to a quadratic equation by multiplying both the sides by 'y' .

$$y \left(3y + \frac{4}{y} \right) = y \ (5)$$

$$\Rightarrow 3y^2 + 4 = 5y$$

$$\Rightarrow 3y^2 - 5y + 4 = 0$$ is a quadratic equation.

Example: (Type - III)

Suppose $\sqrt{25 - x^2} = x - 1$

Now by squaring both the sides we get

$25 - x^2 = x^2 - 2x + 1$

$\Rightarrow 2x^2 - 2x - 24 = 0$

$\Rightarrow x^2 - x - 12 = 0$ is a quadratic equation.

Example: (Type - IV)

Suppose $\sqrt{2x + 3} - \sqrt{x + 1} = 1$

Transfer one of the radicals to R.H.S.

Than we get $\sqrt{2x + 3} = 1 + \sqrt{x + 1}$

Squaring both the sides we get

$2x + 3 = 1 + (x + 1) + 2 \sqrt{x + 1}$

or $2x + 3 = x + 2 + 2 \sqrt{x + 1}$

Retaining term with radical sign an one side only. We get

$$x + 1 = 2 \sqrt{x + 1}$$

Now squaring both the sides again we get

$$(x + 1)^2 = 4 (x + 1)$$

or $x^2 - 2x - 3 = 0$ which is a quadratic equation.

Example: (Type - V)

Suppose $2 (x^2 + \frac{1}{x^2}) - 3 (x + \frac{1}{x}) -1 = 0$

We can write $x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2.$

So the given equation can be written as

$2 (x + \frac{1}{x})^2 - 2 - 3 (x + \frac{1}{x}) - 1 = 0$

or $2 (x + \frac{1}{x})^2 - 3 (x + \frac{1}{x}) - 5 = 0$

Suppose $y = x + \frac{1}{x}$

Then the equation will be $2y^2 - 3y - 5 = 0$

Which is a quadratic equation.

8.11 Solution of problems involving quadratic equations

Some simple problems can be solved by applying quadratic equations whose roots are the solution to the problems. It may happen that out of the two roots only one has a meaning for the problem. Then any root that does not satisfy the conditions of the problem must be rejected.

Example:1: The product of two consecutive positive integers is 306. Find out the integers.

Solution: Suppose the integers are x and $x + 1$.

Then $x(x+1) = 306 \Rightarrow x^2 + x - 306 = 0$

$\Rightarrow (x + 18)(x - 17) = 0 \Rightarrow x = -18$ and 17

But $x = -18$ is rejected.

So the two integers are 17 and 18 (Ans)

Example:-2 The product of Rama's age (in years) five years ago with his age (in years) 9 years later is 15. Find Rama's present age.

Solution: Suppose Rama's present age is x years.

His age was 5 years ago $= x - 5$ years.

His age 9 years later will be $=$ $x + 9$ years.

Therefore $(x - 5)(x + 9) = 15$

(According to the given condition)

Hence $x^2 + 4x - 45 = 15$

or $x^2 + 4x - 60 = 0$ (It is a quadratic equation).

Example: (Type - III)

Suppose $\sqrt{25 - x^2} = x - 1$

Now by squaring both the sides we get

$25 - x^2 = x^2 - 2x + 1$

$\Rightarrow 2x^2 - 2x - 24 = 0$

$\Rightarrow x^2 - x - 12 = 0$ is a quadratic equation.

Example: (Type - IV)

Suppose $\sqrt{2x + 3} - \sqrt{x + 1} = 1$

Transfer one of the radicals to R.H.S.

Than we get $\sqrt{2x + 3} = 1 + \sqrt{x + 1}$

Squaring both the sides we get

$2x + 3 = 1 + (x + 1) + 2 \sqrt{x + 1}$

or $2x + 3 = x + 2 + 2 \sqrt{x + 1}$

Retaining term with radical sign an one side only. We get

$$x + 1 = 2 \sqrt{x + 1}$$

Now squaring both the sides again we get

$$(x + 1)^2 = 4 (x + 1)$$

or $x^2 - 2x - 3 = 0$ which is a quadratic equation.

Example: (Type - V)

Suppose $2 (x^2 + \frac{1}{x^2}) - 3 (x + \frac{1}{x}) - 1 = 0$

We can write $x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2.$

So the given equation can be written as

$2 (x + \frac{1}{x})^2 - 2 - 3 (x + \frac{1}{x}) - 1 = 0$

or $2 (x + \frac{1}{x})^2 - 3 (x + \frac{1}{x}) - 5 = 0$

Suppose $y = x + \frac{1}{x}$

Then the equation will be $2y^2 - 3y - 5 = 0$

Which is a quadratic equation.

8.11 <u>Solution of problems involving quadratic equation:</u>

Some simple problems can be solved by applying quadratic equations whose roots are the solution to the problems. It may happen that out of the two roots only one has a meaning for the problem. Then any root that does not satisfy the conditions of the problem must be rejected.

<u>Example:1</u>: The product of two consecutive positive integers is 306. Find out the integers.

<u>Solution:</u> Suppose the integers are x and $x + 1$.

Then $x(x + 1) = 306 \Rightarrow x^2 + x - 306 = 0$

$\Rightarrow (x + 18)(x - 17) = 0 \Rightarrow x = -18$ and 17

But $x = -18$ is rejected.

So the two integers are 17 and 18 (Ans)

<u>Example:-2</u> The product of Rama's age (in years) five years ago with his age (in years) 9 years later is 15. Find Rama's present age.

<u>Solution:</u> Suppose Rama's present age is x years.

His age was 5 years ago $= x - 5$ years.

His age 9 years later will be $=$ $x + 9$ years.

Therefore $(x - 5)(x + 9) = 15$

(According to the given condition)

Hence $x^2 + 4x - 45 = 15$

or $x^2 + 4x - 60 = 0$ (It is a quadratic equation).

Solving it we get $x = 6$ and $x = -10$.
Since x is present age of Rama it can not be negative. Therefore we reject $x = -10$. So the present age of Rama is 6 years.

8.12 Ruler compass method of solving a quadratic equation:

(i) $x^2 = 3$

Ruler compass method of constructing $\sqrt{3}$ is known to us. That gives the solution of the quadratic equation in (i).

We also know the method of constructing a square equal in area to a given rectangle. The above equation can also be written as $x^2 = 3 \times 1$

This shows that x is the length of the side of the square whose area is equal to the area of a rectangle' have sides of length $\sqrt{3}$ units and 1 unit.

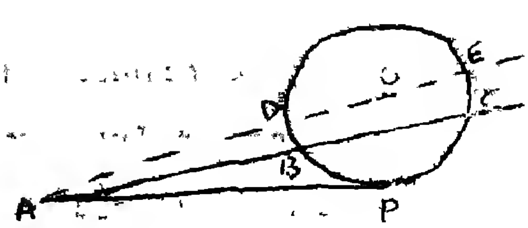
Thus a rectangle of sides of length 3 units and 1 unit is constructed and then a square of equal area is drawn. The length of the side of the square is the solution of the equation given in (i).

(ii) $x^2 + 2x = 5$ i

This equation can be rewritten as

$x\ (x + 2) = (\sqrt{5})^2$

In the figure given alongside AP is the tangent to the circle and AC is a secant. It can be proved that $AP^2 = AB \times AC$.



If $AB = x$, $BC = 2$ and $AP = \sqrt{5}$

then we get $(\sqrt{5})^2 = x\,(x + 2)$

Which is the given equation. Again AC being an arbitrary secant passing through A, we can even choose the secant drawn through the centre 0. As such we get

$$AP^2 = AE \times AD \quad \ldots\ldots.(1)$$

Taking $DE = 2$ and $AD = x$

$$(1) \Rightarrow x^2 = x\,(x + 2)$$

Thus the method of construction is as follows.

<u>1st Step:</u> A circle is drawn with diameter = 2 units and the centre is named as 0.

<u>2nd Step:</u> By construction $\sqrt{5}$ is determined.

<u>3rd Step:</u> Any point on the circle is named as P and 0 the radius $\overrightarrow{OP}$ is drawn. $\overrightarrow{PA}$ is drawn at P such that $\overrightarrow{PA} \perp \overrightarrow{PO}$.

A is marked on $\overrightarrow{PA}$ such that $PA = \sqrt{5}$.

<u>4th Step:</u> $\overrightarrow{PO}$ is drawn, and the point where $\overrightarrow{PO}$ cuts the circle is named as D.

$$PD = x$$

Thus we have algebraic as well as geometrical methods for solving a quadratic equation.

9.1 Ratio, proportion and variation

Very often we feel the necessity of comparing two quantities or measures. Some examples are given below.

(i) Gopal is older than Fayaz i.e. Gopal's age is more than that of Fayaz.

(ii) Gopal is older than Fayaz by 5 years.

(iii) The price of a coloured T.V. is 3 times that of a Black and white one of the same make.

Thus example (i) expressed a general way of comparison where as examples (ii) and (iii) are somewhat specific.

In example (ii), between the ages of two persons we come to know which is more and how much more it is. So in the said comparison between two, one is how much more or less than the other is specified.

In example (iii), between the prices of two T.V.s we come to know how many times is the price of one T.V. as compared to the other and such which is more in price and which is less could also be made out.

So in this comparison between two, one is how many times the other that has been specified.

If the price of a B.W. T.V. is assumed as x rupees, the price of coloured one is $3x$ rupees.

$$\frac{\text{The Price of the coloured T.V.}}{\text{The Price of the B \& W T.V.}} = \frac{3x}{x} = \frac{3}{1}$$

This sort of comparison wherein we come to know also how many times is one quantity of another is known as a ratio.

Definition:

Ratio is a comparison between two non-zero quantities specifying how many times is one quantity of the other.

In example (iii) above we write the comparison of the prices of a coloured T.V. and a B & W one as -

$$\frac{\text{Price of the coloured T.V.}}{\text{Price of the B \& W T.V.}} = \frac{3}{1} \text{ or } 3:1$$

We say, the prices of the coloured and B & W models of T.V.s are in the ratio 3:1 .

Note:- $\frac{3}{1}$ and 3:1 both express a ratio . More than two quantities can also be compared in the form of ratio.

Supposing the monthly income of A is double of that B and the monthly income of B is $\frac{2}{3}$ of that of C.

Assuming the income of C as x Rs.

Income of B becomes $\frac{2}{3}x$ and

Income of A becomes $2 \times \frac{2}{3}x = \frac{4x}{3}$

Thus the incomes of A,B and C are in the ratio

$$x : \frac{2x}{3} : \frac{4x}{3} \quad \text{.........(1)}$$

Had we taken -

C's income as 3x Rs.

B's income would be $\frac{2}{3} \times 3x = 2x$ Rs.

A's income would be $2 \times 2x = 4x$ Rs.

Thus the ratio of the incomes of A,B and C would then be equal to 3x:2x:4x ....(2)

Since the ratios in (1) and in (2) express the comparison of the same three quantities, the ratios are supposed to be equal. Thus

$$x : \frac{2x}{3} : \frac{4x}{3} = 3x : 2x : 4x$$

That is a ratio is not altered by multiplying all the terms of a ratio by the same number (not equal to 0). Hence multiplying the ratio in (2) by $\frac{1}{x}$ we get

$$3x \times \frac{1}{x} : 2x \times \frac{1}{x} : 4x \times \frac{1}{x}$$

$$= 3 : 2 : 4$$

Thus we say that the monthly incomes of A,B and C are in the ratio 3:2:4. Equivalent ratios are obtained all the terms of a ratio by the same number (not equal to zero).

9.2 Some terms related to a ratio of two things:

(i) In the ratio a : b
'a' is known as the anticident and
'b' is known as the consequent.

(ii) b:a is known as the inverse ratio of a:b

(iii) The compared ratio of two ratios a : b and c : d is (a × c) : (b × d)

We may also have the compound ratios of more than 2 ratios determined in the same way as in case of two. Thus - compound ratio of the given ratios

(the product of the anticidents) : (the product of the consequents)

(iv) Duplicate ratio of a:B is the compound ratio of a ; b and itself. Thus -

the duplicate ratio of $a:b = a^2 : b^2$

similarly the triplicate ratio of $a : b = a^3 : b^3$.

(v) Sub-duplicate ratio of p:q is the ratio r:s if the duplicate ratio of r : s is p : q,

$$\text{i.e. } r^2 : s^2 = p : q$$

$$\Rightarrow r : s = \sqrt{p} : \sqrt{q}$$

$\Rightarrow$ sub-duplicate ratio of $p:q = \sqrt{p} : \sqrt{q}$

similarly the sub-triplicate ratio of $p : q$

$$= \sqrt[3]{p} : \sqrt[3]{q}$$

9.3 Comparison of two ratios:

We are conversant with the process of comparing two fractions. As we know, to compare two fractions we equalise their denominators. And also in the same way ordering of any number of fractions can be done.

Since a ratio is just an otherwise form of writing a fraction wherein the numerator becomes the antecident and the denominator becomes the consequent, for comparing ratios . We equalise their consequents . Example follows:

Example:-1 Let us compare the ratios 1:2, 3:5 consequents are 2 and 5. Their L.C.M. = 10 Hence the consequents of each of the two ratios is to changed to 10.

$1:2 = (1 \times 5) : (2 \times 5) = 5 : 10$

$3:5 = (3 \times 2) : (5 \times 2) = 6 : 10$

$3:5 > 1:2$

Thus w en the consequents are made equal, the ratio having the greater anticident is greater than the other having the smaller anticident.

Example:-2 Arrange the ratios 2:3, 3:5 and 7:12 in increasing order.

Solution: Consequents are 3, 5 and 12 , L.C.M. of the consequents = 60

Hence all the consequents are to be changed to 60.

2:3 = (2 x 20) : (3 x 20) = 40 : 60

3:5 = (3 x 12) : (5 x 12) = 36 : 60

7:12 = (7 x 5) : (12 x 5) = 35 : 60

The ratios in increasing order are

7 : 12, 3 : 5, 2 : 3

9.4 Solving problems involving ratios:

When the ratio of two unknowns is given we need not substitute two symbols for two unknowns. Substitution using one unknown can help us solving the problem more easily. Exemple fellows :

Example:- 3 The monthly salaries of Ram and Gopal are in the ratio 5 : 7 and each of them spend Rs.3500.00 out of his salary. What should be their monthly salaries. So that the balances after expenditure are together equal to Rs.5000.00 .

Solution: (Substitution taken for both the unknowns)

Let Ram's salary be x Rs.

According to the question $x : y = 5 : 7$

$$\Rightarrow \frac{x}{y} = \frac{5}{7}$$

$$\Rightarrow 7x = 5y$$

$$\Rightarrow 7x - 5y = 0 \quad \ldots\ldots\ldots (1)$$

Ram's balance money = $(x - 3500)$ Rs.

Gopal's balance money = $(y - 3500)$ Rs.

According to the question -

$$x - 3500 + y - 3500 = 5000$$

$$\Rightarrow x + y = 12000 \quad \ldots\ldots\ldots\ldots (2)$$

Thus we get two equations with two unknowns x and y and solving them simultaneously, the values of the unknowns can be determined.

<u>Solution</u> (using one symbol)

Since the ratio of the salaries of A and D is given as 5:7, we assume -

Ram's salary as 5x Rs.

and Gopal's salary as 7x Rs.

Balance of Ram = $(5x - 3500)$ Rs.

Balance of Gopal = $(7x - 3500)$ Rs.

According to the question -

$$5x - 3500 + 7x - 3500 = 5000$$

$$\Rightarrow 12x = 12000$$

Now we get one equation with one unknown and after solving the equation the value of x can be determined. Thereafter the salaries of Ram and Gopal can be determined.

Thus substitutions for the unknown of which the ratio is given can be taken using one symbol only and thereby the solution becomes confined to only one equation.

Let us see another example involving ratio.

Example:- 4 The ratio of the boys and girls on rolls in a school was 8:5. In the final examination 50 boys and 40 girls passed out. If the boys and girls are in the ratio 5:3, determine number of boys and number of girls that were there originally.

Note: There are two ratios involved in the question. One ratio concerns with the boys and girls which were originally there and the one concerns with the remaining number of boys and girls in the school. The original numbers are related to the remaining numbers. Hence know one of them, the other one be determined.

Hence substitution shall be made either for the numbers of boys and girls that were originally there or the remaining numbers of boys and girls.

Solution: Let original no. of boys be 8x and the no. of girls be 5x, 50 boys and 40 girls passed out. remaining no. of boys $= 8x - 50$ and remaining of girls $= 5x - 40$

But their ratio is 5 : 3

$$\therefore \frac{8x - 50}{5x - 40} = \frac{5}{3}$$

x can be determined by solving the above equation and thereafter the original number of boys and original number of girls can be determined.

9.5 An important concept relating to ratio:

What kind of quantities are those of which we are talking of the ratio ? The examples that have been discussed show that each of the ratios given speaks of the comparison between the quantities (or measures) of the same kind e.g.

In example (3), the given ratio concerns with the salaries of two persons expressed in rupees

In example (4) the given ratio concerns with numbers of boys and girls.

Ratio of the quantities (or measures) of the same kind of thing taken in the same unit of measure gives us a rational number, like -

The ratio of the weights of two objects both in the unit of Kg may be 3:5. This ratio is nothing but a rational number $\frac{3}{5}$.

We also talk of the ratio of two measures with different kinds of unit-measure. For example -

We express the ratio between price and weight of different commodities in the following manner. We say the price of rice is Rs.8.00 per Kg. which means that the ratio of price in rupees and weight in Kg of a certain quantity of rice is 8:1

Speed 50 Km/hr. shows that the ratio of the distance covered and the time required is 50: 1 and so on.

9.6 Relation of ratios with percentage:

Since percentage is also a mode of comparison of one number with another. It can also be expressed as a ratio and vice versa.

(i) When we say 5% what we mean ?

We mean that there are two numbers. The first number contains as many 5s, as the number of 100s in the 2nd number.

Otherwise speaking :-

The first number is 5 when the second number is 100

The ratio of the two numbers = 5 : 100

Hence 5% = 5 : 100 = 1 : 20

(ii) When two numbers are in the ratio 3:5, it means -

When the 2nd number is 5, the 1st one is 3

When the 2nd no. is 1 the 1st one is $\frac{3}{5}$

When the 2nd no. is 100, the 1st one is $\frac{3}{5} \times 100$

The comparison in the two numbers is $\frac{3}{5} \times 100$ in 100 which is equal to 60%

Thus the ratio 3:5 in % $= \frac{3}{5} \times 100$

$= 60\%$

9.7 Usefulness of ratio:

(i) it compares two (or more) measure);

(ii) it helps us working with smaller numbers in place large numbers by taking there ratio instead of the numbers themselves;

(iii) through ratios comparison of more than one quantities can be easily visualised.

9.8 Proportion:

The concept of proportion is rather an extension of ratio.

Definition: Proportion is a statement showing equality of two ratios

When the ratios a:b and c:d are equal then the statement

$$a : b = c : d$$

is said to be a proportion.

We say -

a, b, c and d are proportional or,

a, b, c and d are in proportion.

We also write it as $a:b :: c : d$

We read it as $a:b$ is as $c : d$

In the proportion $a:b = c:d$

a and d are known as the extremes and b and c are known as the means. Expressing the ratios contained in the proportion as fractions, we get -

$$\frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow ad = bc$$

Thus we see that -

The product of the extremes is the product of the means.

9.9 Continued Proportion:

If $a:b = b:c$, we say

a, b and c are in continued proportion.

(or a,b and c are in proportion) In this case 'b' is the mean proportional , a and c are the extremes.

We get -

$$\frac{a}{b} = \frac{b}{c} \Rightarrow ac = b^2$$

that is, product of the extremes = (mean proportional)^2

9.10 Proportions available from a given proportion:

(1) If $a : b = c : d$

then $\frac{a}{b} = \frac{c}{d}$

$$\Rightarrow \frac{1}{\frac{a}{b}} = \frac{1}{\frac{c}{d}}$$

$$\Rightarrow \frac{b}{a} = \frac{d}{c} \Rightarrow b:a = d:c$$

$\therefore$ $a:b = c:d \Rightarrow b:a = d:c$

This property is known as <u>invertendo</u>

(2) If $a:b = c:d$

then $\frac{a}{b} = \frac{c}{d}$

$\Rightarrow ad = bc$

$$\Rightarrow \frac{ad}{cd} = \frac{bc}{cd} \quad [\because c \neq 0,\ d \neq 0]$$

$$\Rightarrow \frac{a}{c} = \frac{b}{d} \Rightarrow a : c = b : d$$

$\therefore$ $a:b = c:d \Rightarrow a:c = b:d$

This property is known as <u>alternendo</u>

(3) If $a : b = c : d$

then $\frac{a}{b} = \frac{c}{d}$

$$\Rightarrow \frac{a}{b} + 1 = \frac{c}{d} + 1 \text{ (adding 1 on both sides)}$$

$$\Rightarrow \frac{a+b}{b} = \frac{c+d}{d} \Rightarrow (a+b) : b = (c+d) : d$$

$\therefore$ $a:b = c:d \Rightarrow (a + b) : b = (c + d) : d$

This property is known as compenondo.

(4) If $a : b = c : d$

then $\frac{a}{b} = \frac{c}{d}$

$\frac{a}{b} - 1 = \frac{c}{d} - 1$ (subtraction 1 from both sides

$\frac{a-b}{b} = \frac{c-d}{d} \Rightarrow (a-b) : b = (c-d) : d$

$a:b = c:d \Rightarrow (a-b) : b = (c-d) : d$

This property is known as <u>dividendo</u>.

(5) If $a : b = c : d$

then $\frac{a+b}{b} = \frac{c+d}{d}$ (by componendo) ...(a)

again $\frac{a-b}{b} = \frac{c-d}{d}$ (by dividendo) .....(b)

$\frac{(a)}{(b)} \Rightarrow \frac{a+b}{b} \times \frac{b}{a-b} = \frac{c+d}{d} \times \frac{d}{c-d}$

$\Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d} \Rightarrow (a+b) : (a-b) = (c+d) : (c-d)$

$a:b = c:d \quad (a+b) : (a-b) = (c+d) \quad (c-d)$

This property is known as componendo and dividendo.

Application on componendo and dividendo.

<u>Example</u>: If $\frac{a}{b} = \frac{2}{3}$ find out the value of $\frac{3a+4b}{3a-4b}$?

<u>Solution</u>:

$\frac{a}{b} = \frac{2}{3} \Rightarrow \frac{3}{4} \times \frac{a}{b} = \frac{3}{4} \times \frac{2}{3}$

$\Rightarrow \frac{3a}{4b} = \frac{1}{2}$

By componendo and dividendo

$\frac{(3a)+(4b)}{(3a)-(4b)} = \frac{1+2}{1-2}$

$\Rightarrow \frac{3a+4b}{3a-4b} = \frac{3}{-1} = -3 : 1$

<u>Example</u>: If $\frac{2a+3b}{2a-3b} = \frac{7}{5}$, find out $\frac{a}{b}$?

Solution: $\frac{2a + 3b}{2a - 3b} = \frac{7}{5}$

$\Rightarrow \frac{(2a + 3b) + (2a - 3b)}{(2a + 3b) - (2a - 3b)} = \frac{7 + 5}{7 - 5}$ (By componendo and dividendo)

$\Rightarrow \frac{4a}{6b} = \frac{12}{2}$

$\Rightarrow \frac{a}{b} = \frac{12}{2} \times \frac{6}{4}$

$\Rightarrow \frac{a}{b} = \frac{9}{1} = 9 : 1 = 9$

Example: If $\frac{4a + 5b}{4a - 5b} = \frac{9}{7}$, find out the value of $\frac{3a + 2b}{3a - 2b}$

Solution: $\frac{4a + 5b}{4a - 5b} = \frac{9}{7}$

$\Rightarrow \frac{(4a + 5b) + (4a - 5b)}{(4a + 5b) - (4a - 5b)} = \frac{9 + 7}{9 - 7}$, (by componendo and dividendo)

$\Rightarrow \frac{8a}{10b} = \frac{16}{2}$

$\Rightarrow \frac{a}{b} = \frac{16}{2} \times \frac{10}{8} = \frac{10}{1}$

$\Rightarrow \frac{3}{2} \times \frac{a}{b} = \frac{3}{2} \times \frac{10}{1}$

$\Rightarrow \frac{3a}{2b} = \frac{15}{1}$

$\Rightarrow \frac{3a + 2b}{3a - 2b} = \frac{15 + 1}{15 - 1}$ = (By componendo and dividendo)

$\Rightarrow \frac{3a + 2b}{3a - 2b} = \frac{16}{14} = \frac{8}{7}$

(6) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots\ldots$

$\Rightarrow \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots\ldots = \frac{a + c + e + \ldots\ldots}{b + d + f + \ldots\ldots}$

$\left(\frac{\text{Sum of the NS}}{\text{Sum of the DS}}\right)$ -

Where $N^{r}S$ stands for numerator, and $D^{r}S$ stands for denominator.

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots\ldots = K$

$\Rightarrow a = bk,\ c = dk,\ e = fk,\ \ldots\ldots$ etc.

Putting these values of a, c, e, ... etc.

We get $\frac{a + c + e + \ldots\ldots}{b + d + f + \ldots\ldots} = \frac{bk + dk + fk + \ldots\ldots}{b + d + f + \ldots\ldots}$

$= \frac{K\,(b + d + f + \ldots\ldots)}{(b + d + f + \ldots\ldots)} =$

$K = \frac{a}{b} = \frac{c}{d} = \frac{e}{f}\ \ldots\ldots$

__Example:__ If a, b, c, d are in continued proportion then their reciprocals are also in continued Proportion and vice versa.

__Solution:__

a, b, c, d are in continued proportion

$\Rightarrow \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = K$ (say)

$\Rightarrow \frac{1}{a/b} = \frac{1}{b/c} = \frac{1}{c/d} = \frac{1}{K}$

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \frac{1}{K}$

$\Rightarrow \frac{(\frac{1}{a})}{(\frac{1}{b})} = \frac{(\frac{1}{b})}{(\frac{1}{c})} = \frac{(\frac{1}{c})}{(\frac{1}{d})} = \frac{1}{K}$

$\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$ and $\frac{1}{d}$ are in continued proportion.

Converse can be proved by taking $\frac{1}{a}$ in place of a, $\frac{1}{b}$ in place of b, $\frac{1}{c}$ in place of c and $\frac{1}{d}$ in place of d.

__Example:__ If $(1 + x)$, $(4 + x)$ and $(5 + x)$ are in continued proportion then find out the value of x ?

Solution: Since $(1 + x)$, $(4 + x)$, $(5 + x)$ are in continued proportion.

$$\Rightarrow \frac{1 + x}{4 + x} = \frac{4 + x}{5 + x}$$

$$\Rightarrow \frac{(1 + x) + (4 + x)}{(1 + x) - (4 + x)} = \frac{(4 + x) + (5 + x)}{(4 + x) - (5 + x)},$$

(by componendo and dividendo)

$$\frac{5 + 2x}{-3} = \frac{9 + 2x}{-1}$$

$$\frac{(5 + 2x)}{3} = \frac{(9 + 2x)}{1} \Rightarrow 3(9 + 2x) = 1(5 + 2x)$$

$$\Rightarrow 27 + 6x = 5 + 2x \Rightarrow 6x - 2x = 5 - 27$$

$$\Rightarrow 4x = -22 \Rightarrow x = \frac{-22}{4} = \frac{-11}{2} = -5\frac{1}{2}$$

The value of $x = -5\frac{1}{2}$

Example: If $x = \frac{6ab}{a+b}$, find out the value of

$$\frac{x + 3a}{x - 3a} + \frac{x + 3b}{x - 3b} \;?$$

Solution:

$$x = \frac{6ab}{a+b}$$

$$\Rightarrow \frac{x}{3a} = \frac{2b}{a+b}$$

$$\Rightarrow \frac{x + 3a}{x - 3a} = \frac{2b + (a + b)}{2b - (a + b)},$$ (By componendo and and dividendo)

$$\Rightarrow \frac{x + 3a}{x - 3a} = \frac{a + 3b}{b - a} \quad \text{..........(1)}$$

Again $x = \frac{6ab}{a + b}$

$$\Rightarrow \frac{x}{3b} = \frac{2a}{a+b}$$

$$\Rightarrow \frac{x + 3b}{x - 3b} = \frac{2a + (a + b)}{2a - (a + b)},$$ (By componendo and dividendo)

$$\Rightarrow \frac{x + 3b}{x - 3b} = \frac{3a + b}{a-b} \quad \text{..........(ii)}$$

From (i) and (ii) we have

$$\frac{x + 3a}{x - 3a} + \frac{x + 3b}{x - 3b} = \frac{a + 3b}{b - a} + \frac{3a + b}{a - b}$$

$$= \frac{- a - 3b + 3a + b}{a - b}$$

$$= \frac{2a - 2b}{a-b} = \frac{2\ (a - b)}{a-b} = 2$$

....

VARIATION

10.1 Introduction:

Quantities which can assume any value. What, So-ever, it is known as a variable. If two variables are such that the change in the value of one causes the other one to change so that the ratio between them under the original conditions and the changed conditions as well remain equal. They are said to vary directly with each other. For example, the weight of an article and the price of it.

Weight of sugar	Price	Ratio
1 Kg.	Rs.9.00	1:9
5 Kg.	Rs.45.00	5:45 = 1:9
8 Kg.	Rs.72.00	8:72 = 1:9

and, so on. Thus we see that $w : p$ has the same ratio in all cases.

If two variables are such that each varies as the reciprocal of the other, then they are said to vary inversely with each other. The following is an example.

When the number men is increased, the number of days required for completion of a work is decreased and vice-versa.

No. of men employed to do a work (n)	Time required for completion (t)
20	12 ....(1)
10	24 ....(2)
30	8 ....(3)

It can be seen that -

in (1), $n : \frac{1}{t} = 20 : \frac{1}{12} = 240 : 1$

in (2), $n : \frac{1}{t} = 10 : \frac{1}{24} = 240 : 1$

in (3), $n : \frac{1}{t} = 30 : \frac{1}{8} = 240 : 1$

Thus n varies as $\frac{1}{t}$

It can also be seen that -

in (1), $t : \frac{1}{n} = 12 : \frac{1}{20} = 240:1$

in (2) : $t : \frac{1}{n} = 24 : \frac{1}{10} = 240 : 1$

in (3), $t : \frac{1}{n} = 8 : \frac{1}{30} = 240:1$

Thus t varies as $\frac{1}{n}$. We can also see that -

in (1), $n \times t = 20 \times 12 = 240$

in (2), $n \times t = 10 \times 24 = 240$

in (3), $n \times t = 30 \times 8 = 240$

Thus in each of the cases the product of $n \times t$ remains unaltered.

If x varies as y (i.e. varies directly as y), we write - $x \propto y$

and we have seen that

$$\frac{x}{y} = K \text{ (a constant)}$$

or $x = K\,y$

If x varies inversely as y, that is, x varies as $\frac{1}{y}$, we write -

$$x \propto \frac{1}{y}$$

and we have seen that

$$x \times y = K \text{ (a constant)}.$$

It can be seen that the unitary method of calculation is very much analogous to variation. The illustrations below will make it clear.

Example:-1 If 300 m^2 of land can be turfed by 18 men in 1 day, then

(i) how many man can turf 400 m^2 in 1 day ?

(ii) what area of land can be turfed by 30 men in one day ?

10.2 Unitary method of solution:

(i) 300 m^2 requires 18 men

1 m^2 will require $\frac{18}{300}$ men .....(1)

400 m^2 will require $\frac{18}{300}$ X 400

$= 18 \times \frac{400}{300}$ ....

$= 24$

It can be seen that number men and the area to be turfed vari directly with each other.

No. of men (n)	Area to be turfed(a)
$n_1 = 18$	$a_1 = 300$
$n = ?$	$a_2 = 400$

10.3 Solution using principle of variation:

Since $n \propto a$

$n = K \times a$ ....(1) where K is a constant

When $n = 18$, $a = 300$

Substituting the above values in the preceeding equation we get

$18 = K \times 300$

or $K = \frac{18}{300}$ ....(2)

When $a = 400\ m^2$

$(1) \Rightarrow n = K \times a$

$= \frac{18}{300} \times 400$

$= 24$

We can observe the value of K obtained in step (2) of variation process is the same as the no. of men required for turfing $1\ m^2$ area as determined in step (1) of the unitary method.

So the analogy is established. Similarly solving Ex. 1 (ii) -

Unitary Method:

18 men are required for $300\ m^2$

1 man is required for $\frac{300}{18}\ m^2$

30 men are required for $\frac{300}{18} \times 30$ or $500\ m^2$

Variation method:

$a \propto n \Rightarrow a = k\,n$ ....(1)

When $a = 300,\ n = 18$

$(1) \Rightarrow 300 = k \times 18$, $K = \frac{300}{18}$

when $n = 30$

$(1) \Rightarrow a = K \times n = \frac{300}{18} \times 30 = 500\ m^2$

It can further be seen that the value of the constant K in the variation method is exactly the same as the area that can be turfed by 1 man in 1 day.

Thus the analogy in the two methods can be well established.

Example:-2 24 men can do a work in 40 days How many men can do the same work in 15 days ?

Unitary Method:

40 days are required for 24 men

1 day is required for 24 x 40 men

15 days are required for $\frac{24 \times 40}{15} = 64$ men

Variation: The number of men (m) required for doing a work varies inversely as the time (t) they take.

$$m \propto \frac{1}{t}$$

$$\Rightarrow m = K \times \frac{1}{t} \quad \ldots\ldots\ldots(1)$$

When $m = 24$, $t = 40$

$$(1) \Rightarrow 24 = K \times \frac{1}{40}$$

$$\Rightarrow K = 24 \times 40 \quad \ldots\ldots\ldots(2)$$

when $t = 15$

$$(1) \quad m = K \times \frac{1}{t}$$

$$= (24 \times 40) \times \frac{1}{15}$$

$$= 64$$

It can be seen that the value of the constant K as determined in step (1) of the variation methdd is the same as the no. of men required for doing the work in 1 day as calculated in the unitary method.

10.4 Alternative form of the Variation values:

(i) $x \propto y \Rightarrow x = k\,y$ ....(1)

Supposing when $x = x_1$, $y = y_1$

then (1) $x_1 = k\,y_1$ ....(2)

again when x and y change from x_1 to x_2

and y_1 to y_2 then (1) $\Rightarrow x_2 = ky_2$...(3)

$\frac{(2)}{(3)} \Rightarrow \frac{x_1}{x_2} = \frac{y_1}{y_2}$ ....(4)

$\Rightarrow \frac{x_1}{y_1} = \frac{x_2}{y_2}$ ....(5)

This can also be used as a rule for direct variation.

(ii) $x \propto \frac{1}{y} \Rightarrow x = \frac{k}{y} \Rightarrow xy = K$ ......(1)

Supposing when $x = x_1$, $y = y_1$

then (1) $\Rightarrow x_1 \times y_1 = K$ ....(2)

Again when x changes from x_1 to x_2

and y changes from y_1 to y_2

then (1) $\Rightarrow x_2 \times y_2 = K$ ....(3)

(2) and (3) $\Rightarrow x_1\,y_1 = x_2\,y_2$....(4)

$\Rightarrow \frac{x_1}{x_2} = \frac{y_1}{y_2}$ ....(5)

Thus step (4) in (i) and step (4) in (ii) show the alternative forms for the rules of direct variation and inverse variation respectively, step (5) of (i) and step (5) of (ii) are also the alternative forms for the rules of direct variation and inverse variation respectively.

Exemples follows:

Exemple - 3 Find the final volume of a gas when its temperature increases from 200° K to 300°K at constant pressure. The initial volume of the gas at 200° K is 250 ml.

Solution: Since at constant pressure the volume of a gas is directly proportional to the temperature so these two are in direct variation. So applying the direct variation principle i.e.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$V_2 = \frac{T_2}{T_1} V_1 = \left(\frac{300}{\cancel{200}} \times \cancel{250}^{125} \right)$$

$$= 375 \text{ ml}$$

Exemple - 4 What is the length of the shadow of a tree of length 17 m. when its length was 15 m., when its shadow was 20 m.

Solution: The length of the shadow of a tree increases as the length increases or vice versa.

Let length of the tree be T_1 and T_2 and its corresponding shadows are b_1 and b_2

So applying the direct variation principles

$$\frac{T_1}{b_1} = \frac{T_2}{b_2} \qquad b_2 = \frac{T_2}{T_1} \times b_1$$

$$= \frac{17}{\cancel{15}} \times \cancel{20} = \frac{68}{3} \text{ m}$$

Exemple- 5. At constant temperature the volume of a given mass of any gas is inversely proportional to the pressure. If P_1, P_2 and V_1, V_2 is the pressure and the volumes of a gas at constant pressure are given at the different stages then according to the principle

$$V_1 P_1 = V_2 P_2$$

$$\Rightarrow V \propto \frac{1}{P} \text{ when 1 is constant}$$

Exemple What is the volume of a gas at constant temperature when its pressure is 315 ml. and given that the volume of the same gas is 840 ml. when its pressure is 270 ml.

Solution Applying the inverse variation principle i.e. $V_1 P_1 = V_2 P_2$ and putting the known values we have

$$270 \times 840 = 315 \times V_2$$

$$\Rightarrow V_2 = \frac{270 \times 840}{315} \text{ ml.}$$

$$= 720 \text{ ml.}$$

10.5 Joint Variation:

There are occasions when one variable varies depending on more than one variables. Examples follow.

(i) Number of days required for doing a work varies depending on the number workers and the duration work rendered every day.

(ii) Volume of a given mass of gas varies depending upon the absolute temperature and the pressure.

There is a theorem relating to such situations. The theorem states -

If A varies as B when C is constant, and A varies as C when B is constant, then A varies and BC when both B and C vary.

Proof: Suppose a, is the value of A when b_1 and c_1 are the values of B and C respectively. Further suppose, that a_2 is the value of A when b_2 and c_2 are the values of B and C respectively.

The change of A from a_1 to a_2 under two situations, e.g.

(1) B changes from b_1 to b_2 c remaining unchanged at c_1

(2) C changes from C_1 to C_2 B remaining unchanged at b_2 . We represent the above facts below. The value of A becomes

= a_1 when B and C have the values b_1 and c_1 respectively .... (1)

$= a'$, when b_2 becomes the value of B and c_1 continues to be value of C ......(ii)

$= a_2$, when b_2 continues to be value of B and c_2 becomes the changed value of C .....(iii)

From (i) and (ii) we get

$$\frac{a_1}{a'} = \frac{b_1}{b_2} \quad \ldots\ldots\ldots\ldots \text{(iv)}$$

From (ii) and (iii) we get

$$\frac{a'}{a_2} = \frac{c_1}{c_2} \quad \ldots\ldots\ldots \text{(v)}$$

Hence (iv) x (v) $\frac{a_1}{a'} \times \frac{a'}{a_2} = \frac{b_1}{b_2} \times \frac{c_1}{c_2}$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \times \frac{c_1}{c_2}$$

Thus $A \propto B \times C$ (proved)

The following example may be seen .

Example:-6 8 men can do a work in 30 days working 6 hours a day. In what time can 10 men do the work if they work for 8 hours a day ?

Solution: (Unitary method)

8 men working 6 hours a day to the work in 30 days.

1 man working 6 hours , 30 x 8 days

1 man working 1 hr. a day does the work in 30 x 8 x 6 days.

10 men working 1hr. a day can do the work in $\frac{30 \times 8 \times 6}{10}$ days.

10 men working 8 hrs a day can do the work in $\frac{30 \times 8 \times 6}{10 \times 8}$ days = 18 days.

10.6 Variation principle:

No. of days required (n) varies inversely as the duration of work done every day (t). i.e. $n \propto 1/t$ ....(i)

No. of days (n) varies inversely as the no. of men (m) employed i.e $n \propto \frac{1}{m}$ .... (ii)

(i) and (ii) $\Rightarrow$ $n \propto \frac{1}{t} \times \frac{1}{m}$

$\Rightarrow$ $n = K \times \frac{1}{tm}$ ...... (1)

When $m = 8$, $n = 30$, $t = 6$

Eq.(1) $\Rightarrow$ $30 = K \times \frac{1}{6 \times 8}$

$\Rightarrow$ $K = 30 \times 6 \times 8$ ....(ii)

When $m = 10$ and $t = 8$

Eq. (1) $\Rightarrow$ $n = K \times \frac{1}{t\ m} = 30 \times 6 \times 8 \times \frac{1}{8 \times 10}$

$= 24$

It can be seen that the values of K in the solution or variation principle is the same as the number of days required by 1 man working for one hour a day.

Thus closeness between the method of variation and unitary method can be observed. Examples follow.

Example:-7 5 persons spend 1000 rupees in 30 days. What amount of money will be spent by 7 persons in 20 days ?

Solution: (Data is shown in a table for convenience)

No.of person (n)	Expenditure (m)	No.of days (t)
5	1000	30
7	?	20

Expenditure (m) varies directly with no. of persons (n)

i.e. $m \propto n$ .... (1)

Expenditure (m) varies directly with the time (t)

i.e. $m \propto t$ ....(2)

(1) and (2) $\Rightarrow m \propto n \times t \rightarrow m = K \times n \times t$ ...(3)

when $n = 5$, $m = 1000$ $t = 30$

(2) $\Rightarrow 1000 = K \times 5 \times 30$

$\Rightarrow K = \frac{1000}{5 \times 30} = \frac{20}{3}$

When $n = 7$, $t = 30$

(3) $\Rightarrow m = K \times n \times t$

$= \frac{20}{3} \times 7 \times 30 = 1400$ Rs. (Ans.)

Example:-8 Combined gas law

$V \propto \frac{1}{p}$, (T is constant) by Boyle's law

$V \propto T$, (P is constant) by Charle's law

$\Rightarrow V \propto \frac{1}{p}, T$, (When P and T both vary)

$\Rightarrow V = K \frac{T}{P}$

$\Rightarrow \frac{PV}{T} = K$ constant

$\Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2\,V_2}{T_2}$

Example: For a given mass of any gas at constant temperature 200°K the pressure is 100° m mercury and the volumes 300 ml. Find the temperature of the given gas if its pressure changes to 400 ml mercury and volume changes to 100 ml.

Solution: From the joint variation principle wes known that

$$\frac{P_1\ V_1}{T_1} = \frac{P_2\ V_2}{T_2}$$

Putting the value we get

$$T_2 = \frac{P_2\ V_2}{P_1\ V_1} \times T_1$$

$$= \frac{400 \times 100}{100 \times 300} \times T_1$$

$$= \frac{4}{3} \times T_1 = \frac{4}{3} \times 200^{\circ}\ K$$

$$= \frac{800^{\circ}}{3}\ K = 266\ \frac{2}{3}^{\circ}\ K$$

This can be expressed in degree centgrade scale.

as $t^{\circ}\ C = (t + 273)^{\circ}\ K$

Exp. $50^{\circ}\ C = (50 + 273)^{\circ}\ K$

$= 323^{\circ}\ K$

So while conversing a Kelvin scale in to centigrade scale we have to substract 273 from the given value so that the result will be in centigrade scale.

GEOMETRY

POINT

11.1 Introduction:

The concept of point is as old as human civilization. In different contexts we have been assuming different objects as points.

11.2 'Point' in different contexts:

1. The P.E.T. of a school gives two marks on the play ground for volley ball or badminton posts by a spade. These represent points.

2. On a globe or geographical map Bhubaneswar and Calcutta are marked with dots. But each of them occupies a vast area. In a diagram of the solar system even earth and the sun are shown as dots. But each one is enermous in size.

3. Even in certain situations it is not possible to denote some objects in the form of a point. To locate the Solar system as a point in the Spiral Nebula as an impossible task.

'Point' is the basic concept in Geometry. We can however have a good idea of these concepts by considering examples. A fine dot made by a sharp pencil on a sheet of paper resembles a geometrical point very closely than a point on the black board by a piece of chalk. The sharper the needle of the pencil the closer is the dot to the concept of a geometrical point.

Euclid, the father of Geometry, defined point in his own way in his treatise 'Element' . Later on it was observed that the definition had two major defects:

(i) the definition was based on a negative statement.

(ii) it was circular in nature

Whatever definitions for 'Point' have been proposed, these have been found to be defective and wanting. Safer method is to take the term 'Point' undefined and then make adequate number of axioms about point to yeild the geometry we desire.

L I N E

12.1 Concept:

1. Fold a piece of paper and press the two parts together. On unfolding it you will see that a straight crease is formed which resembles a line.

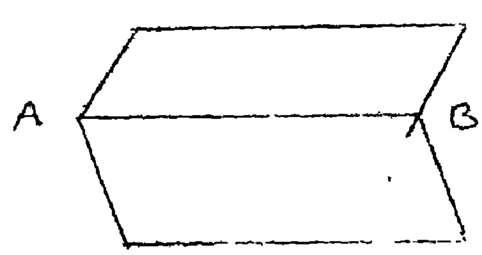


Fig.1

2. Hold a piece of thin spring by its two ends and pull it, so that the spring becomes tight and straight. That is part of a line.

Fig.2

3. The edge of a table, a box and the side of a rectangle are examples of a portion of a line.

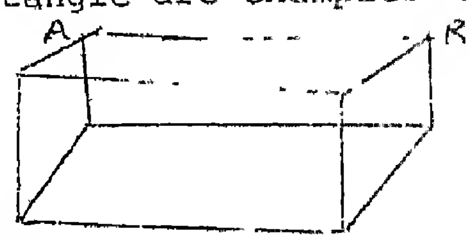


Fig.3

4. A line means the line in its totality and not a portion of it. Obviously a line cannot be drawn or shown wholly on a piece of paper whatever may be the size. In practice only a portion of a line is drawn and arrow - heads are given in both the ends to indicate that it extends indefinitely in both the directions.

Fig.4

So a line has no end points.

In different times men of Mathematics have also taken attempts to define line. Afterwards these definitions were also found to be defective. So 'line' has been accepted as an undefined term and adequate axioms have been developed to clarify the concept of a 'line'.

PLANE

13.1 Concept

1. Let us discuss about different surfaces of cylinder, cone, and a cube.

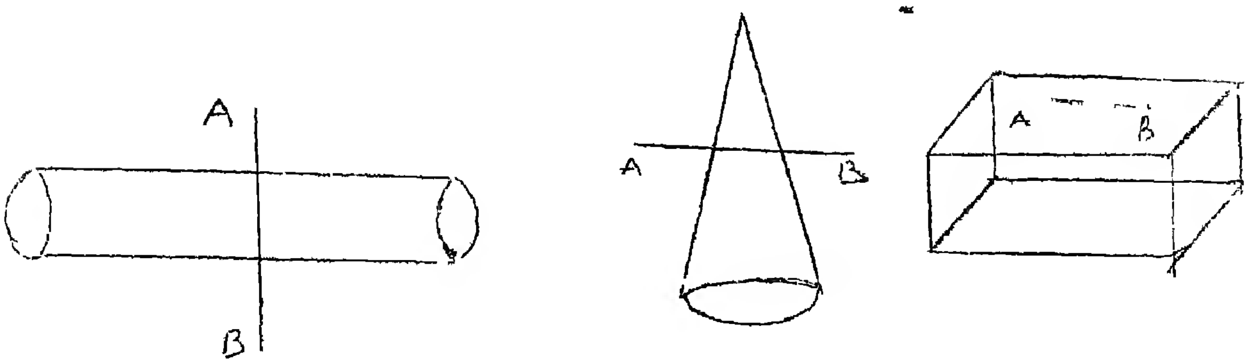


Fig.1

Take a pencil AB. Put that pencil on the curved surface of the cylinder as shown in the figure-I. The pencil will not remain stable. Similarly on the curved surface of the cone, the pencil will not be stable. The pencil will remain stable when put on the surface of a cube.

So it is now clear that the pencil remains stable as more points of the pencil touch the surface of the cube. Such surface is part of a plane.

2. If we insert three nails of different length on a piece of small wooden plate and keep the plate upside down, these three nails will touch the plane.

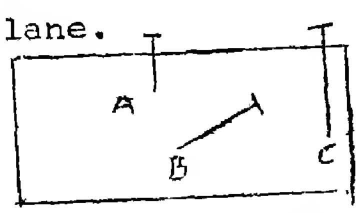


Fig.2

Hence three non-co-llinear points will always determine a plane.

3. A curved rod will be straight when it touches a plane surface at every point.

Fig.3.

4. The surface of a smooth wall or the smooth surface of a black board is the geometrical plane which extends endlessly in all directions.

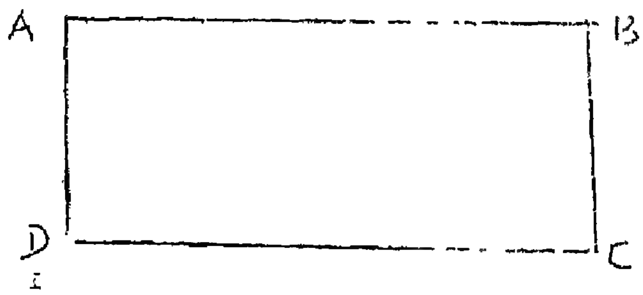


Fig.4

With these initiative ideas, we build the axioms regarding plane taking it as undefined term.

13.2 THE MAIN COMPONENTS OF AXIOMATIC GEOMETRY

1. Undefined terms

   a) The terms having no definition are called undefined term.

   b) The number of undefined terms should be less.

Examples Point, Line, Plane, and Space
(Space is not mentioned as undefined term in our text books since at the secondary level we are confined to plane Geometry only).

2. Postulates or Axioms

The basic facts which are taken for granted without proofs are called axioms. The axioms or postulates express our intuitive ideas about how points, lines and planes are related.

Axiom - 1

1. Lines and planes are set of points.
2. Given two points there is exactly one line containing them.
3. A plane contains at least two points and if a plane contains two given points it contains the line through these points.
4. There exists exactly one plane containing three given non-collinear points.
5. If two planes intersect, their intersection is a line.

13.3 Examples from daily life:

Axiom - 2 We can know that the line has straightness. Straightness can be seen from the experience of the wood cutter. He first takes two points at the end of the timber. He stretches a blackened thread through above two points. He presses the blackened thread on the timber. Then he gets the line to cut the timber.

So it is now clear that through two given points one and only one line can be drawn.

Axiom - 3 We can observe the work of a mason who uses mortar (mixture of cement sand and water) to make the surface of a floor plane. He first makes two small heaps of short height of mortar. He puts his plane stick (Gaj) on the heaps. By water level he tests that the stick is in horizontal position. Then he puts mortar in between these two heights and fill the gap between them.

So it is quite evident that if two points lie on a plane, a line drawn through these two points also lies on that plane.

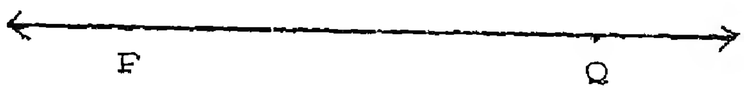
Fig. 5

Axiom - 4 The mason after preparing two heaps as discussed above, prepares the third heap which is not collinear with the first two. Then using his stick he fills the gaps between the three heaps and makes that portion of the floor plane.

So now it is clear that one and only one plane exists containing any three non-collinear points.

Axiom - 5 The intersection of two adjacent walls is part of a line. Hence the intersection of two planes is a line.

<u>Axiom - 6</u> Given any two points P & Q (Not necessarily distinct) there is a unique non-negative real number associated with them. This is called the distance between P and Q.



This distance is denoted by PQ.

<u>Example</u>

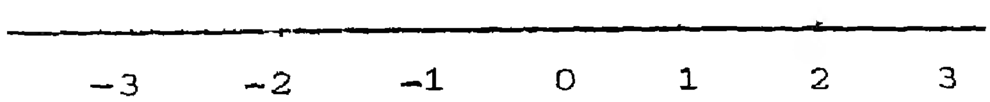
(a) The ariel distance between Keonjhar and Bhubaneswar is 200 Kms. It is a real number.

(b) $P = Q \Longrightarrow PQ = 0$

So the distance between Keonjhar and itself is 0 (Zero).

<u>Axiom - 7</u> There is a one-to-one correspondence between the points of a line and the set of all real numbers such that the distance between any two points is the absolute difference of the corresponding real numbers.

<u>Examples</u>:-



1. (a) We know earlier that line is a set of points.

   (b) These points are related to real numbers i.e. there is one-to-one correspondence between these points and the real numbers.

Let us start from Bhubaneswar to Keonjhar .
On the way we will come across Killometre stone marked with 0, 1, 2 etc. These stones are related to that place like our set of points on the line with real numbers.

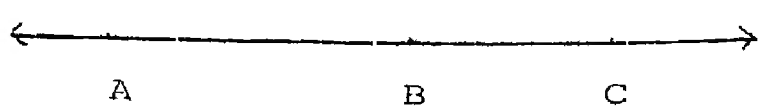
2. The real numbers so related to a point is called the co-ordinate.

3. If we want to find out the distance between two points , it is necessary to take the absolute value of the difference of their co-ordinates.

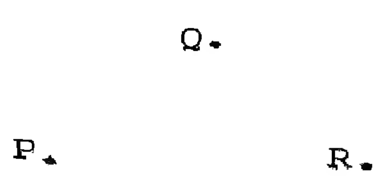
If the co-ordinates of A & B are 5 and 11 respectively $AB = |5 - 11| = |-6| = 6$
or $AB = |11 - 5| = |6| = 6$
If the co-ordinate of A is 'x' and that of B is 'y' , $AB = |x - y|$ $(x, y \in R)$

4. We know that in the set of real numbers there are infinite numbers. Each one of them is related to a point on the line. So a line contains infinite number of points.

13.4 BETWEENNESS



(a) In this figure point B lies in between A and C.



(b) In this figure it is clear that no point lies in between the other two.

Definition:

Three points A, B & C are in a line and $AC = AB + BC$.

We can say that the point B lies in between A and C. This can be indicated

$$A - B - C \Longleftrightarrow C - B - A$$

The co-ordinates of A, B and C are x,y and z respectively. If $x < y < z$ or $x > y > z$ then we can say the point B is in between A and C.

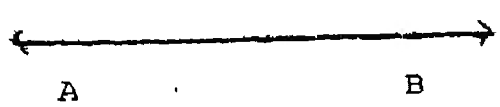
13.5 SEGMENT

A and B are two different points. The union of the set of points between A and B and A, B is called the line segment AB.

We denote the line segment AB by $\overline{AB}$.

So $\overline{AB} = \{A, B\} \cup \{P \mid A - P - B\}$

RAY

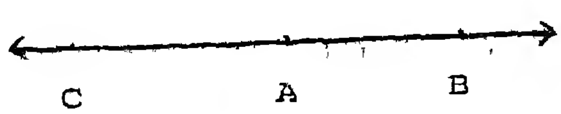


$\overrightarrow{AB} = \overline{AB} \cup \{P \mid A - B - P\}$

and point A is called the vertex of the ray.

13.6 OPPOSITE RAYS

If A lies between B and C then the two rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are said to be opposite rays.



$$\overrightarrow{AB} \cup \overrightarrow{AC} = \overleftrightarrow{BC}$$

13.7 THE RELATION AMONG LINE SEGMENT, RAY AND LINE

(1) $\overline{AB} \subset \overrightarrow{AB} \subset \overleftrightarrow{AB}$

(2) $\overrightarrow{AB} \cup \overrightarrow{BA} = \overleftrightarrow{AB}$

(3) $\overrightarrow{AB} \cap \overrightarrow{BA} = \overline{AB}$

(4) $\overline{AB} \cup \overleftrightarrow{AB} = \overleftrightarrow{AB}$

(5) $\overline{AB} \cap \overleftrightarrow{AB} = \overline{AB}$

Many such relations among $\overline{AB}$, $\overrightarrow{AB}$ and $\overleftrightarrow{AB}$ can be established.

14.1 Planes and their Separation

Introduction

We have intuitive notion that any point on a line divides or separates the line into two parts, taking the point with each part. We get two opposite rays with that point as the common initial point.

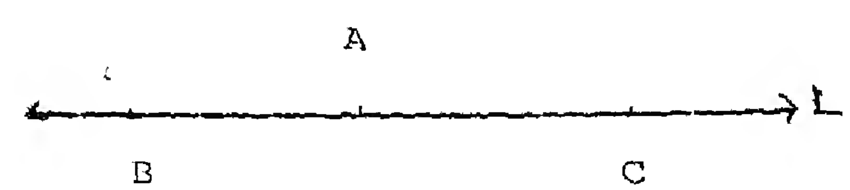


Fig.1

In figure (1) 'A' is a point on a line L. Taking two other points B and C such that B-A-C two rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are obtained. Hence, (i) $\overrightarrow{AB} \cup \overrightarrow{AC} = L$ and (ii) $\overrightarrow{AB} \cap \overrightarrow{AC} = \{A\}$.

A similar situation occurs in case of a plane.

14.2 Plane:- Separation

Let us take a piece of paper. Draw a line on the paper. Then the line divides the plane containing the piece of paper into two separate parts. In other words the line divides the plane into two disjoint sets. Now the plane itself is divided into three separate parts i.e. Half plane H_1, Half plane H_2 and the line L.

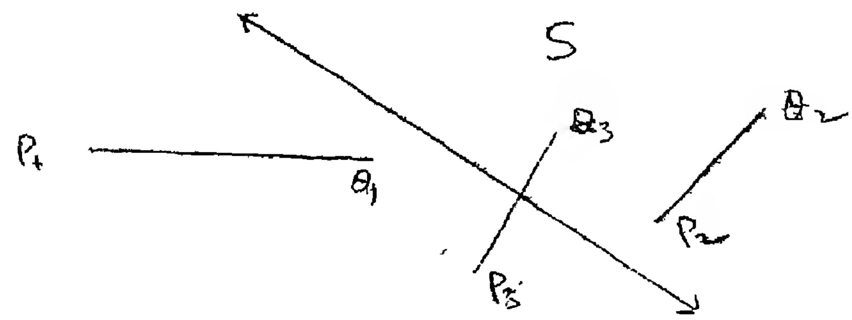


Fig. 2

In figure (2) Consider the plane 'S', the line L divides the plane S into two separate parts as H_1 and H_2. Each of these parts is calleda half plane. Since $H_1 \cap L = \emptyset$, $H_2 \cap L = \emptyset$ and $H_1 \cap H_2 = \emptyset$, H_1, H_2 and L are disjoint sets. But $H_1 \cup H_2 \cup L = S$.

If two points P and Q lie on one side of L, then the line segment PQ does not intersect L. On the other hand, if these two points are in the opposite sides of L , then the line segment $\overline{PQ}$ intersects the line L.

14.3 Connexity of sets:

Given a set S. We take arbitrarily any two points A, B in S and if the entire segment $\overline{AB}$ lies in S, then 'S' is called a convex set.

In Fig.3 the diagrams shown are examples of convex sets.

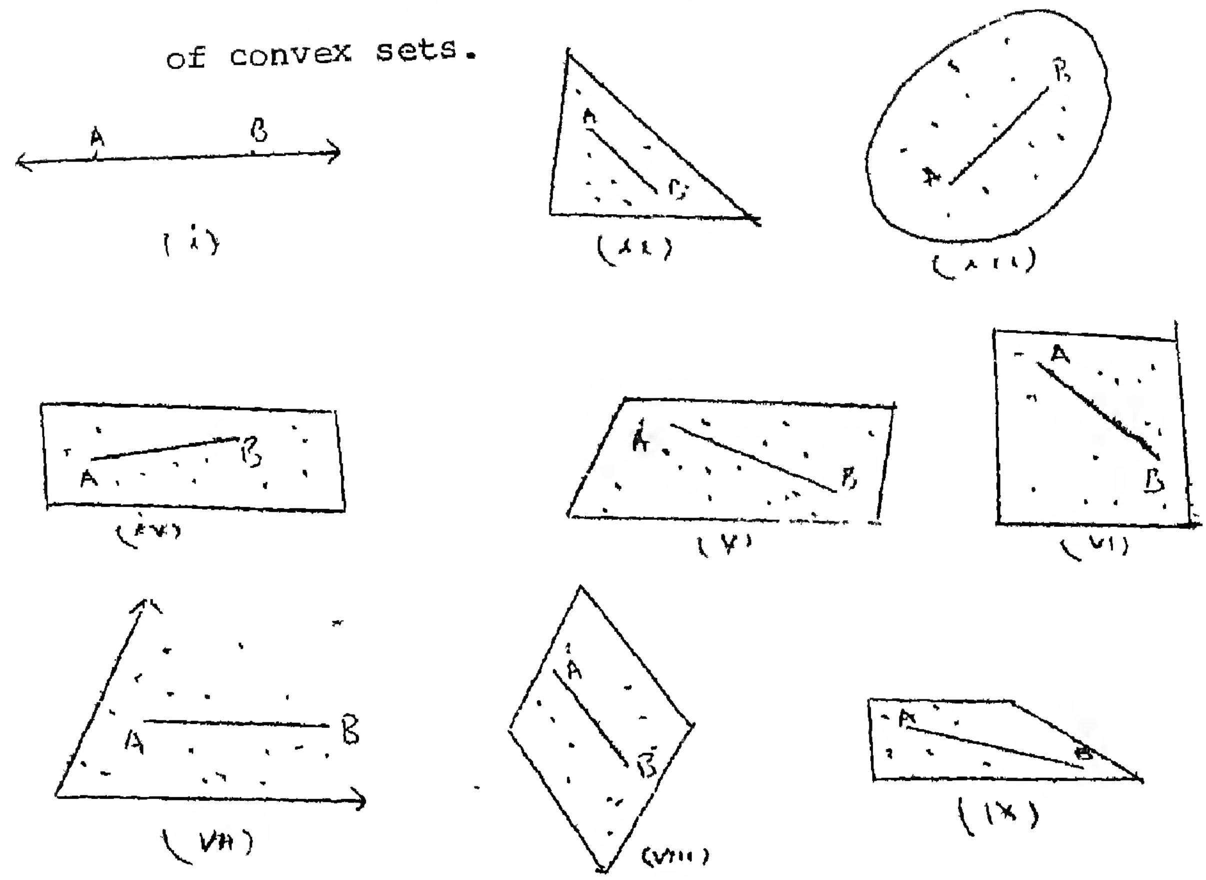


Fig 3

The following sets shown in fig.4 are not convex sets.

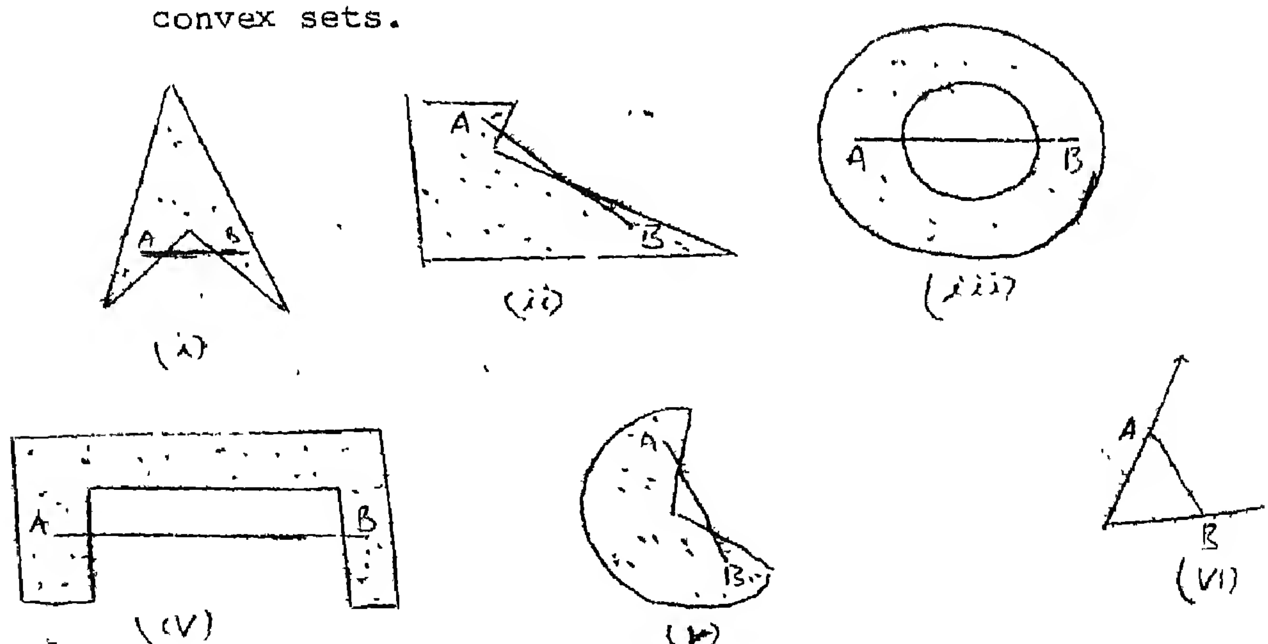


Fig.4

To show the sets in Fig.4 are not convex, we have to arbitraily taken any two points A and B in S and then observe that AB does not wholly lie in S.

<u>Note:-</u> A set consisting of only one point is a convex set as we can take only one point. (say A) in the set and $\overline{AA}$ belong to set. Similarly a line and a space is convex set.

<u>Definition:</u>

S is a set of points. A and B any two points in the set S if $\overline{AB} \subset S$, then S is called a convex set.

(i) $\cap$ of two convex sets is a convex set

(ii) $\{A\}$, $\emptyset$ are convex sets.

<u>Note:-</u> Let us take S_1 and S_2 both being sets of points.

If A and B are arbitrarily any two points in $S_1 \Rightarrow \overline{AB} \subset S_1$ and S_1 is a convex set (1)

Similarly if A, B are two points in S_2,

$\Rightarrow \overline{AB} \subset S_2$ and S_2 is a convex set. (2)

(1) and (2) $\Rightarrow S_1 \cap S_2 = \overline{AB}$

$\Rightarrow \overline{AB} \subset S_1 \cap S_2$

$\Rightarrow S_1 \cap S_2$ is also a convex set.

Hence we conclude that intersection of two convex sets is also a convex set.

Cor. 1 L_1 and L_2 are convex sets. If they intersect at a point P, then

$L_1 \cap L_2 = \{P\}$ is also a convex set.

Cor.2. $L_1 \parallel L_2 \Rightarrow L_1 \cap L_2 = \emptyset$ (Convex)

Since L_1 and L_2 are convex set their intersection $\emptyset$ is also a convex set.

....

ANGLE AND ITS MEASUREMENT

15.1 Introduction

In our day-to-day life we use the term 'angle' in different contexts by refering to the 'angle in a room' we mean that part of the room where two walls meet. Angle of a table means the part of the table where two edges of the top of the table meet. Thus the term 'angle' has emerged from some intuitive ideas associated with different objects.

To create a visual illustration of an angle we take a point A on a piece of paper. Two rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are drawn through A not lying in one line. These two rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ form a figure called an angle.

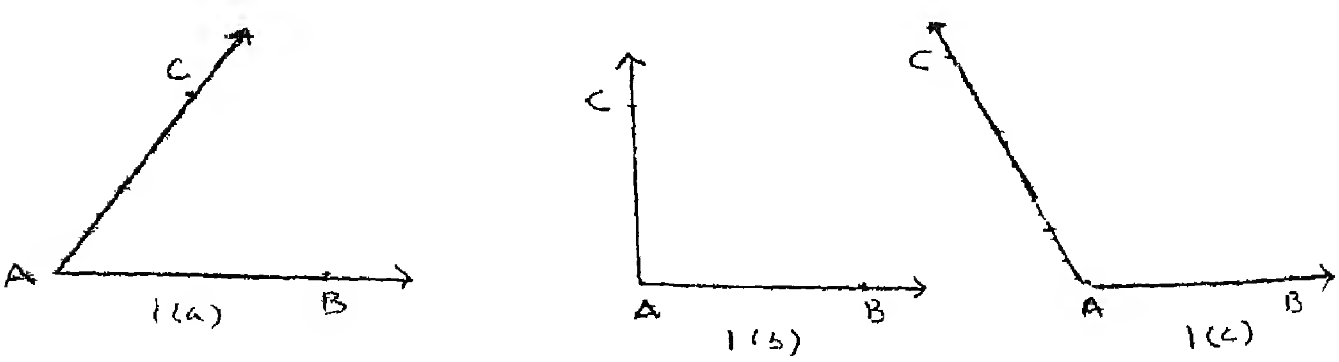


Fig. 1

In fig.1 AB and AC form angles in different positions.

Definition:

"An angle is a figure formed by two rays which have the same end point but do not lie in the same line".

Or

"An angle is a figure formed by the union of two non-collinear rays with a common end point".

Such verbal definitions without a diagram are not so easy to comprehend. So the modern trend has been to give figurative definition as given below.

Definition:

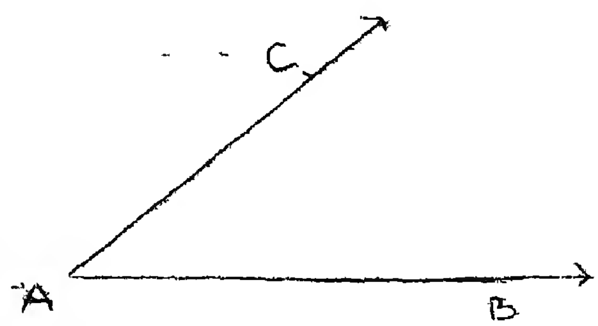


Fig.2

A, B and C are three non-collinear points. The union of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is an angle BAC and is denoted by $\angle BAC$ or $\angle CAB$. With set notations we can also write it in short.

"A, B, C are three non-collinear points

$\Rightarrow \angle BAC = \overrightarrow{AB} \cup \overrightarrow{AC}$."

15.2 Parts of an angle:

The rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are called the arms of the angle BAC.

Vertex: The common end point of the arms of the angle is called its vertex. So A is the vertex of $\angle BAC$.

Sometimes we denote the angle BAC with the help of its vertex as $\angle A$. But it creates confusion when two or more angles have a common vertex. In such a case angles are denoted by numbers as $\angle 1$, $\angle 2$ etc. (fig. 3).

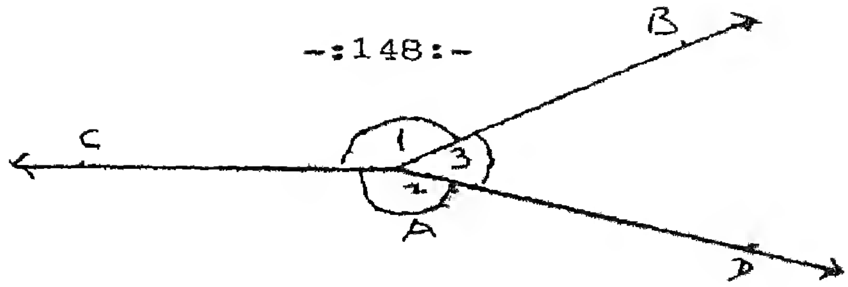


Fig.3

If D is a point on $\overrightarrow{AB}$ different from A and B and E is a point on $\overrightarrow{AC}$ different from A and C of $\angle BAC$, we say that $\angle BAC = \angle DAE$ as $\overrightarrow{AB} = \overrightarrow{AD}$ and $\overrightarrow{AC} = \overrightarrow{AE}$.

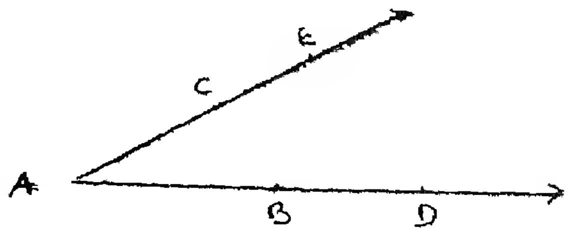


Fig. 4

In fig. 4 we can also say $\angle BAC = \angle BAE = \angle DAC = \angle DAE$

15.3 Interior of an angle:

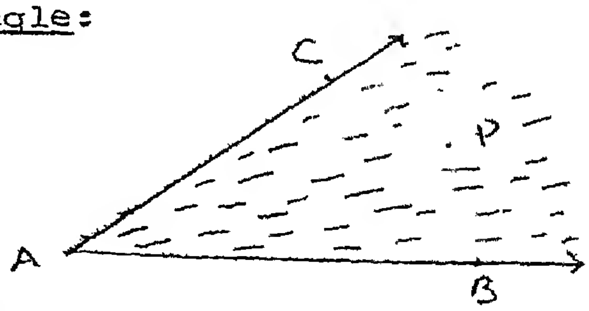


Fig. 5

The interior of $\angle BAC$ is the set of all points lying in the intersection of C-side of $\overleftrightarrow{AB}$ and B-side of $\overleftrightarrow{AC}$. (Fig.5).

A point P lying in the interior of the angle BAC is called the interior point of $\angle BAC$.

15.4 Exterior of an angle:

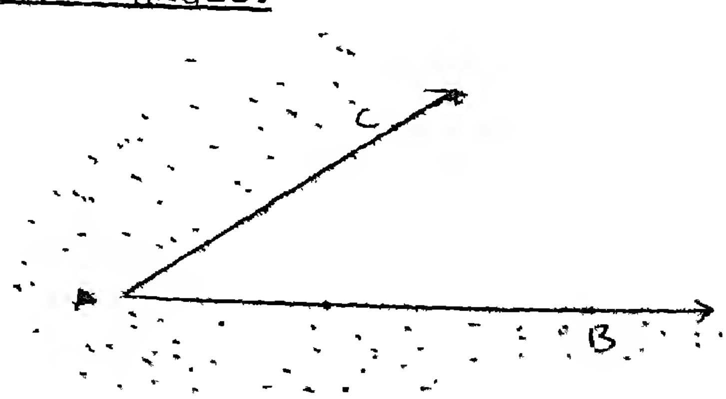


Fig. 6

The exterior of $\angle BAC$ is the set of all points in its plane which do not lie on it or in its interior. (Fig.6)

A point Q lying in the exterior of $\angle BAC$ is called the exterior point.

Notice that -

(i) $\angle BAC$, its interior and its exterior forms three disjoint sets.

In other words

a) $\angle BAC \cap$ interior of $\angle BAC = \emptyset$

b) $\angle BAC \cap$ exterior of $\angle BAC = \emptyset$

c) interior of $\angle BAC \cap$ exterior of $\angle BAC = \emptyset$

(ii) $\angle BAC \cup$ interior of $\angle BAC \cup$ exterior of $\angle BAC$ = the plane containing the angle BAC.

If P and Q are any two arbitrary points in the interior of $\angle BAC$ then $\overline{PQ} \subset$ the interior of $\angle BAC$. Hence the interior of $\angle BAC$ is a convex set. What about $\angle BAC$ and the exterior of $\angle BAC$? Are they convex sets? The answer is 'No'. Why?

13.5 Measure of an angle:

Every angle has a measure. We use a protractor graduated in degrees to measure an angle just as we use a graduated ruler or a tape to find the length of a line segment. The measure of $\angle BAC$ is denoted by as $\angle BAC$.

15.6 <u>Angle Measure Axiom:</u>

The measure of an angle in degrees is a real number 'Y' lying between 0 and 180.
For example : if m $\angle BAC$ = r^{0} then $0 < r < 180$. (Fig.7).

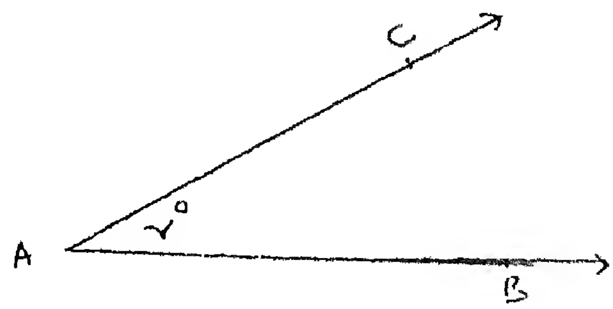


Fig 7

As in the case of $\overline{AB}$ and AB the first one is a set and the second is the real number associated with it so also $\angle BAC$ is a set whereas m $\angle BAC$ is the real number associated with that set.

The concept of an angle-measure is not only important for knowing the measure of an angle but also in classification of angles in the traditional way as right angle, acute angle and obtuse angle.

15.7 <u>Angle Construction Axiom:</u>

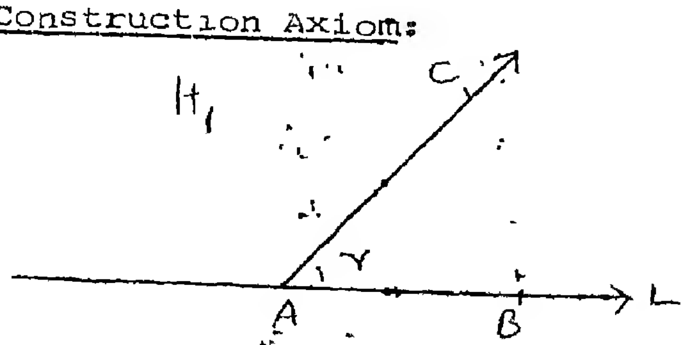


Fig.8

If L is a line, A and B are two points on it and r is a real number between 0 and 180 ($0 < r < 180$) we can draw one and only one ray $\overrightarrow{AC}$ with C lying in the half plane H_1 such that m $\angle BAC$ = r (fig.8).

This axiom states that to every real number between 0 and 180. There corresponds a unique ray in H, while the angle measurement axiom states that to every ray in H_1 there corresponds one and only one real number between 0 and 180.

Thus the above two axioms (angle measure axiom and the angle construction axiom) have been used to graduate the protractor which is used to measure an angle. Besides that now we are in a position to state another axiom to assert that the sum or difference of measures of two angles is the measure of another angle.

15.8 <u>Angle Addition Axiom:</u>

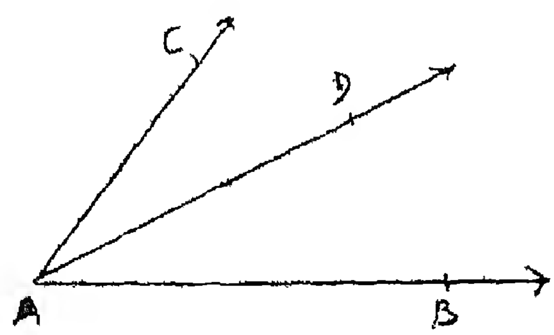


Fig.9

If D is a point in the interior of $\angle BAC$, (fig.9) then $m\angle BAC = m\angle BAD + m\angle DAC$

Hence $m\angle BAD = m\angle BAC - m\angle DAC$

or $m\angle DAC = m\angle BAC - m\angle BAD$.

This axiom helps us in defuising the complementary and supplementary angles. On the basis of this axiom we can also define the angle bisector.

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10.9 Angle Bisector

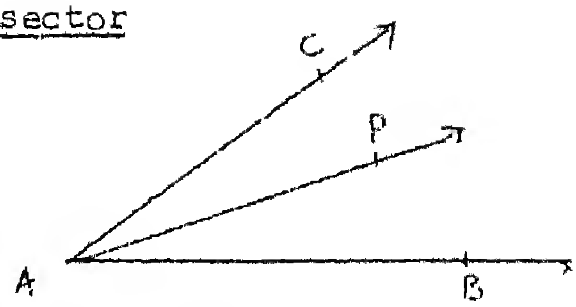


Fig.10

Definision: Given an angle $\angle BAC$ (fig.10) a ray $\overrightarrow{AP}$ is said to be the bisector of $\angle BAC$ if P is a point in the interior of $\angle BAC$ and m $\angle BAP$ = m $\angle PAC$.

From this definition it follows immediately that if $\overrightarrow{AP}$ is the bisector of $\angle BAC$ then m $\angle BAP$ = m $\angle PAC$ = $\frac{1}{2}$ m $\angle BAC$.

15.10 Some Angle Relations:

From the discussions so far we know that an angle is a figure formed by two rays having the same vertex. A figure, in general, may have more than two rays or lines and hence more than one angle. The angles in some figures may possess certain relations among themselves as follows.

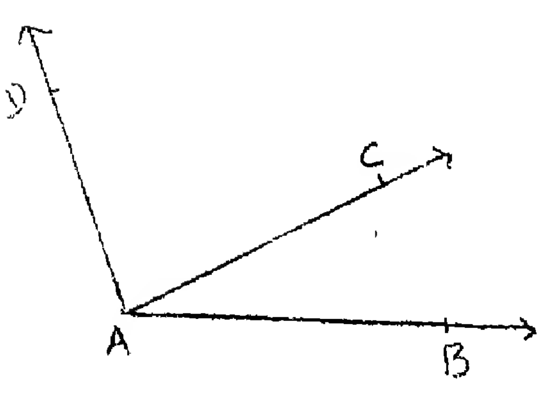


Fig.11

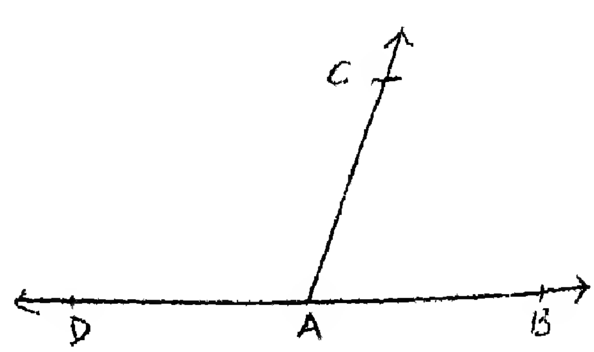


Fig.12

15.11 <u>Adjacent Angles</u>: In the fig.11 there are three rays $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ having a common vertex A. These three rays form two angles $\angle BAC$ and $\angle CAD$. They have a common arm $\overrightarrow{AC}$. The arms $\overrightarrow{AB}$ and $\overrightarrow{AD}$ lie on opposite sides of $\overleftrightarrow{AC}$ with their vertex lying on $\overleftrightarrow{AC}$. Such angle we call adjacent angles.

<u>Definition</u>: Two angles are called adjacent angles if

i) They have the some vertex,

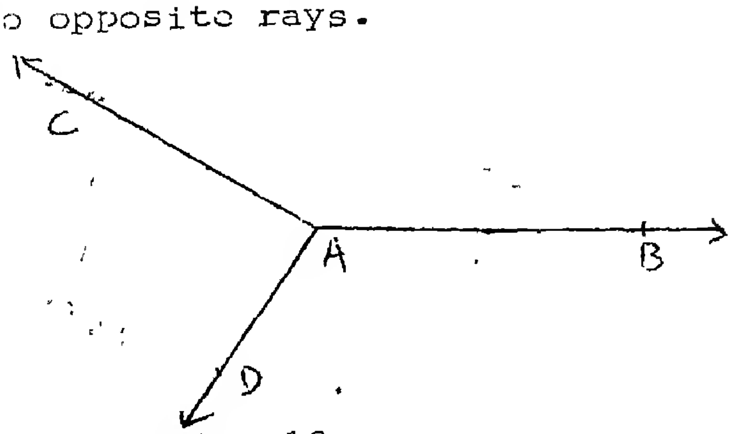
ii) They have a common arm and

iii) The other arm of one angle is on one side of the common arm, and that of the other is on the opposite side.

15.12 <u>Linear Pair of Angles</u>:

In fig.12 $\angle BAC$ and $\angle CAD$ are also adjacent angles but their non-common arms $\overrightarrow{AB}$ and $\overrightarrow{AD}$ are two opposite rays, in other words they are collinear. We call these angles as linear pair of angles.

<u>Definition</u>: Two adjacent angles are said to form a linear pair of angles if their non-common arms are two opposite rays.



<u>Fig. 13</u>

Now let us consider the case of the angles in fig.13. There are three angles $\angle BAC$, $\angle CAD$ and $\angle DAB$. Which of them are adjacent angles ?

Obviously there are three pairs of adjacent angles such as:

i) $\angle BAC$, $\angle CAD$

ii) $\angle BAC$, $\angle BAD$ and

iii) $\angle CAD$, $\angle DAB$

The aforesaid discussions lead us to arrive at the following conclusion with respect to the measures of adjacent angles:

The sum of measures of two adjacent angles is a real number greater than 0° and less than 360°".

16.1 Parallel lines Intersecting Lines:

Introduction:

Given two distinct lines L and M is there a point that lies on both L as well as M ? How many such points are there ?

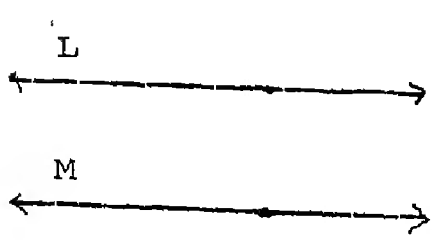


Fig.1

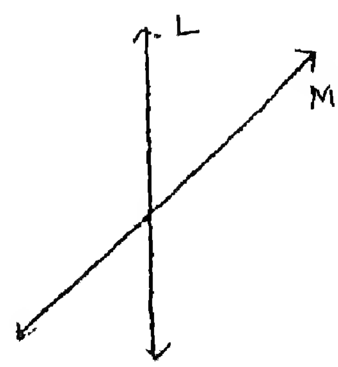


Fig.2

It is obvious that the two lines L and M may or may not have a common point. Question arises that whether it is possible for them to have more than one common point. It is sure that the answer is definitely 'No'.

What happens if we suppose that they have two common points. This will lead us to a contradiction.

Def:- Two lines L_1 and L_2 in a plane are said to be parallel ($L_1 \parallel L_2$) if either $L_1 = L_2$ or if $L_1 \neq L_2$ then $L_1 \cap L_2 = \emptyset$.

Note:- (i) Lines not intersecting each other may not be parallel if they do not lie in the same plane. Such lines are called <u>Skew Lines</u>. Skew lines have no point in common and are not co-planer.

(ii) Two segments are said to be parallel if the lines that contains them are parallel. Thus $\overline{AB} \parallel \overline{CD}$ if $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

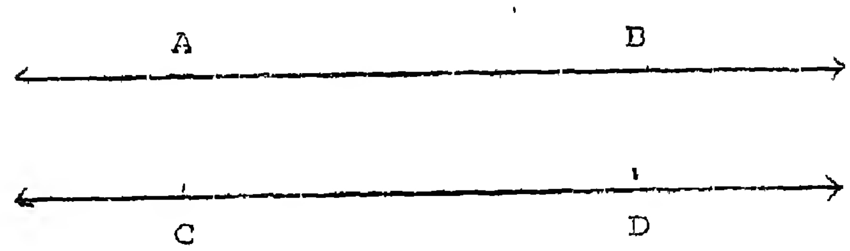


Fig.2(a)

$\overline{AB}$ and $\overline{CD}$ are called parallel segments.

iii) Parallel Rays

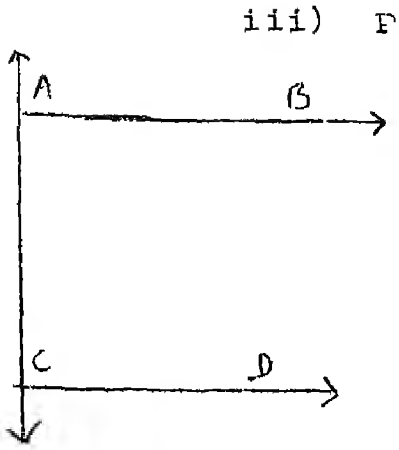


Fig.2(b)

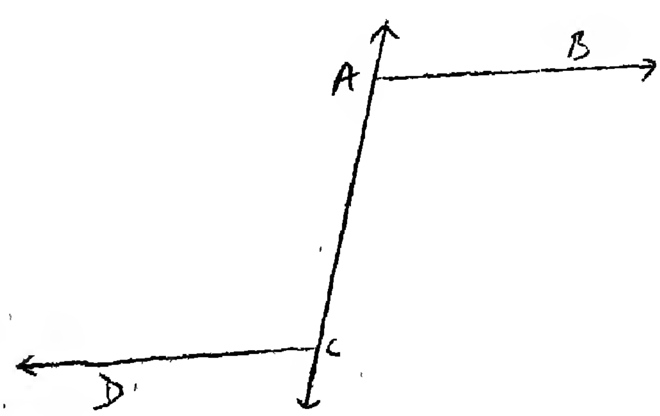


Fig.2(c)

Two rays $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are said to be parallel if $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

Axiom:-

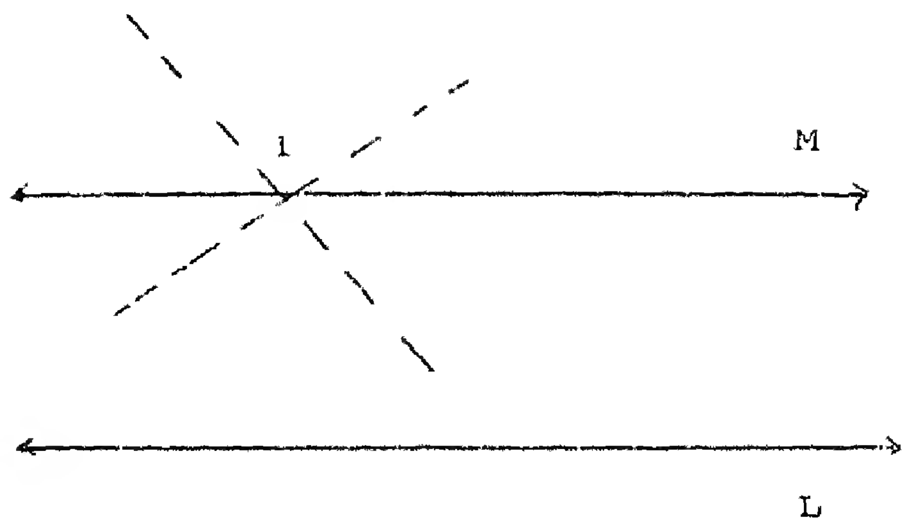


Fig.3

If L is a line and 1 is a point not on L, there is one and only one line, passing through the point '1' and is parallel to L. (Fig.3).

Remarks:- The parallel axiom asserts two facts as follows:-

i) There is a line through a point '1' which is parallel to L and

ii) There is only one such line.

The second part is also stated in an alternative form, called "Play Fairs Axiom, which states:-

Two intersecting lines can not both be parallel to a same line.

iii) It is proved that two lines which are parallel to the same line are parallel to each other.

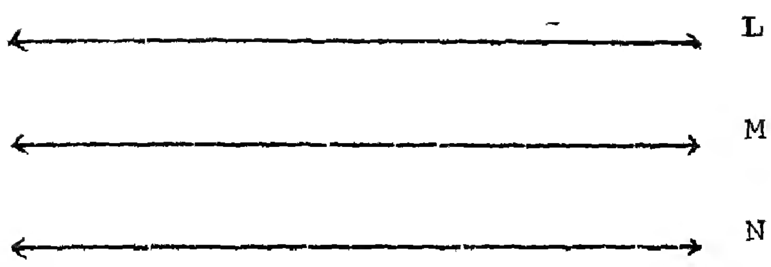


Fig.4

L || M, M || N $\Rightarrow$ L || N

16.2 Angle made by a Transversal with Two Lines

A line which intersect two or more given lines at distinct points, is called a 'transversal' of the given lines in Fig.5

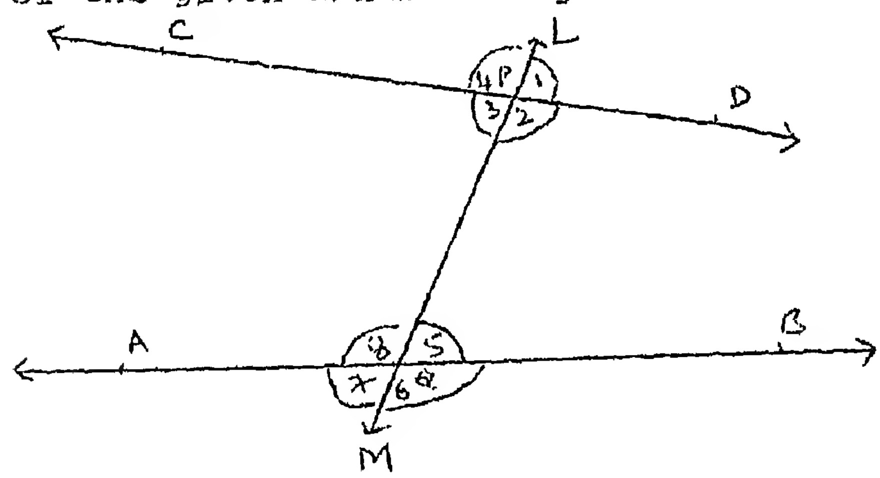


Fig. 5

In figure 5 , $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two lines and a transversal $\overleftrightarrow{LM}$ intersect them in P and Q. Now it is found that there are four angles at point P and also at Q there are four angles. Some of them can be paired together as follows:-

(i) (a) $\angle 1$ and $\angle 5$

(b) $\angle 4$ and $\angle 8$

(c) $\angle 2$ and $\angle 6$

(d) $\angle 3$ and $\angle 7$

These pairs of angle are called pairs of corresponding angles.

(ii) (a) $\angle 3$, and $\angle 5$ | These pairs of angles are called pair of alternative interior angles.
(b) $\angle 2$, and $\angle 8$

(iii) (a) $\angle 2$ and $\angle 5$ | These pairs of angles are called pairs of interior angles on the same side of the transversal.
(b) $\angle 3$ and $\angle 8$

(iv) (a) $\angle 1$ and $\angle 6$ | These pairs of angles are called pairs of exterior angles on the same side of the transversal.
(b) $\angle 4$ and $\angle 7$

When the lines are parallel i.e. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, there exist some very useful relations between the angles of each pair.

When a transversal intersects two parallel line, then the following relations found as follows:-

(i) Each pair of corresponding angles are equal.

(ii) Each pair of alternative interior angles are equal.

(iii) Each pair of consequtive exterior angles are supplementary.

(iv) Each pair of consecutive interior angles are supplementary.

We have number of intersecting and useful relationss between angles. But it is difficult to prove them without coming across two parallel lines for the first time and the earlier theorems were about intersecting lines, we need a new axiom. We agree to have the following as axiom:-

16.3 Axiom:- (Corresponding Angles Axiom)

If a transversal intersects two parallel lines, then each pair of corresponding angles are equal. Conversely if a transversal intersects two lines making a pair of corresponding angles equal, then the lines are parallel.

With the help of this axiom, we can prove other theorems regarding paralleosim .

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LOCI AND CONCURRENCY

17.1 Objectives

By studying this module we shall be able to

i) Define locus

ii) Understand the condition or set of conditions that determine a locus.

iii) Identify the locii determined by different conditions.

iv) Apply the locus theorem to prove theorems on concurrency.

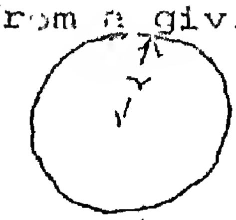
17.2 What we need for defining a locus

i) A geometrical condition or conditions

ii) A set of points obeying that condition.

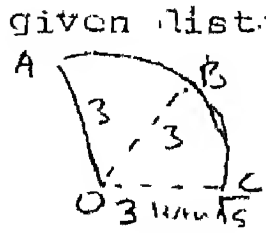
For example:

(a) A sphere is the set of points equidistant from a given point.



A spherical ballon

(b) 'O' is the given point. A circle is the set of points lying in the same plane at a given distance from 'O'.



Let A,B,C be the points on an arc ABC with centre O and radius 3 units. In this case all points on $\overset{\frown}{ABC}$ are at a distance of 3 units from O but all points which are at a distance of

3 units from O are not on $\overset{\frown}{ABC}$. Hence this set can not be said as the locus under the above condition.

For its completeness we want a set containing all points that satisfy the conditions. Its completeness implies two basic concepts:-

(i) Every point of the set satisfies the condition.

(ii) Every point satisfying the given conditions belong to the set.

Hence circle 'C' is the locus with centre 'C' and 3 units as its radius.

17.3 Proof of theorem on Locus:-

The statement of a locus theorem is completely different from the statement of other theorems. It will be clear if we discuss this point with examples

(i) Sum of lengths of any two sides of triangle is greater than the length of the third one.

(ii) Congruent chords of a circle are equidistant from its centre.

In example (1) we are comparing two real numbers namely (i) sum of lengths at two sides and (ii) the length of the third side.

In example (2) two congruent chords of a circle are two compare their distances from the centre.

Therefore in both cases we are comparing the real numbers or showing them to be equal.

Let us take the theorem:-

The locus of points equidistant from two fixed points in the perpendicular bisector of the segment joining these two points.

In this theorem we are going to establish equality of two sets, such as (a) The set of points (s) equidistant from two fixed points and (b) the perpendicular bisector (L) of the line segment joining the two given points.

To prove S = L two steps are necessary.

(i) $X \in S \Rightarrow X \in L$ and

(ii) $X \in L \Rightarrow X \in S$

$S = L \Rightarrow$ SCL and LCS

<u>Note</u>:- It will not be out f the place to quote what has been said by Encyclopedia Britanica in describing the word 'Locus'. It isas under.

'Latin word' 'Locus' means place "In plane Geometry it is the curve (including a straight line) that contains all points in the plane that satisfy a given condition and that contains no points that do not satisfy the given condition".

This makes it very clear that even a traditional presentation the term 'Locus' used to mean a collection of points having two characteristics, which are

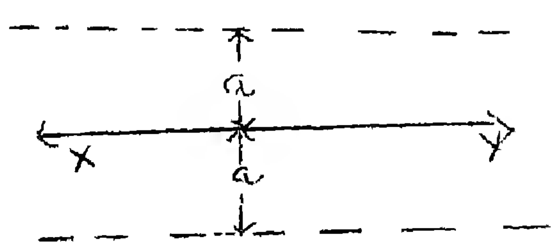
(i) the collection contains those points which satisfy the given condition.

(ii) it does not contain any point which does not satisfy the given conditions.

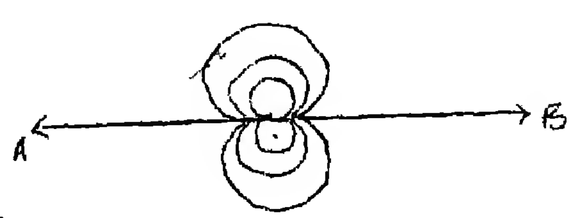
Thus locus is most appropriately a set of points. Since the word 'path' is nowhere defined in geometry, it is not appropriate to use that word for locus.

17.4 Some basic loci

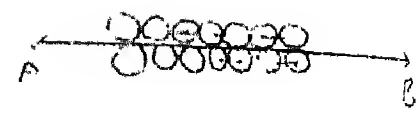
(i) Locus of points lying at a distance of 'a' units from a given line $\overleftrightarrow{XY}$ is a pair of lines each parallel to $\overleftrightarrow{XY}$ lying at a distance 'a' units from $\overleftrightarrow{XY}$ on either side of it.



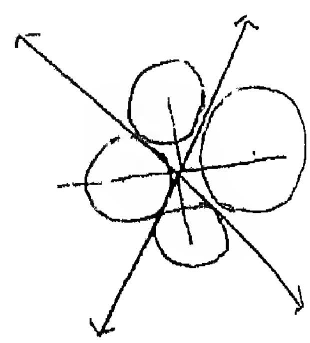
(ii) Locus of the centres of circles which touch a given line $\overleftrightarrow{AB}$ at a given point P on it is a line perpendicular to $\overleftrightarrow{AB}$ at P.



(iii) Locus of centres of circles of a given radius 'r' which touch a given line $\overleftrightarrow{AB}$ follows from no.(i) above.

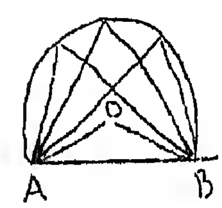


(iv) Locus of the centres of circles which touch two intersective straight lines is a pair of lines bisecting the angles between the given lines.



(v) Locus of points at which a given line segment $\overline{AB}$ makes angles of a given measure (ω) is an arc of the circle with O as centre and OA as radius where

$m\angle OBA = m\angle OBA = \frac{180 - 2\theta}{2}$



(a) Major arc is the locus if $\theta < 90$

(b) Minor arc is the locus if $\theta > 90$.

Thus the proof is complete with proving the proposition and its converse giving one way proof makes it incomplete. With analogy to "if and only if" conditions the proof is to be given both ways.

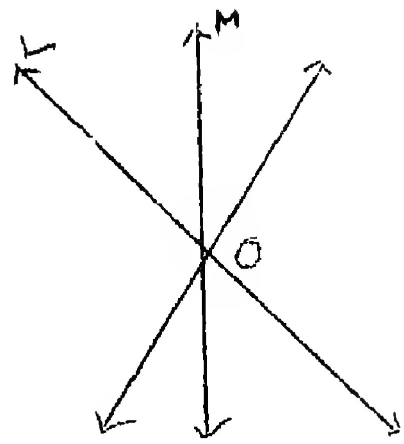
Therefore there are two phases in the proof of a theorem on locus . In the first phase we prove (i) above and in the 2nd phase we prove (ii).

<u>Method-I</u> To prove that every point on the figure satisfies the given condition or conditions. Also to prove that every point that satisfies the given condition or condition lies on the figure.

<u>Method-II</u> To prove that every point on the figure satisfies the given condition or conditions. Also to prove that no point not lying on the figure satisfies the given condition or conditions.

<u>Concurrency</u>

If three or more than three lines pass through a single point, those are called concurrent lines.



The common point is called the point of concurrence of the lines. In the given figure lines L, M, N are concurrent lines and O is the point of concurrency.

<u>Examples:</u>

(i) A triangle has three perpendicular bisectors of its sides . Those are concurrent.

(ii) A triangle has three medians which are also concurrent.

(iii) A triangle has three medians which are concurrent.

For proving these theorems on concurrency we apply the locus theorems.

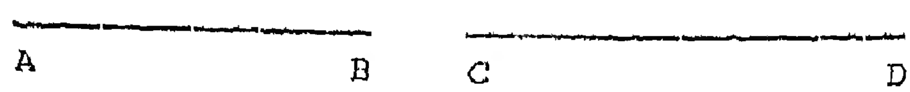
CONGRUENCE

18.1 Congruence of line Segments:

Given two line segments one may be longer (or shorter) than the other or they may be of same size. In the second case, we say they are congruent. The idea of congruence of line segment is best expressed as follows:-

1) Two lines intuitively $\overline{AB}$ and $\overline{CD}$ are congruent if the trace-copy of one can be super imposed on the other so as to cover it completely and exactly.

If line segment $\overline{AB}$ is congruent to line segment $\overline{CD}$, we write $\overline{AB} \cong \overline{CD} \Leftrightarrow AB = CD$



18.2 Congruence Relation in the set of all Line Segments:

The method of super imposition formulates the following properties of conguence of line segments.

(i) $\overline{AB} \cong \overline{CD} \Rightarrow \overline{CD} \cong \overline{AB}$

(ii) $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF} \Rightarrow \overline{AB} \cong \overline{EF}$

(iii) $\overline{AB} \cong \overline{AB}$

18.3 Congruence of two angles:

$\angle ABC$ and $\angle DEF$ are said to be congruent, when $m\angle ABC = m\angle DEF$

The units of measurement of angles can be in terms of degree or grade or circular (radian) measures.

18.4 Congruence of triangles:

While classifying geometrical figures, we use the idea of congruence. Two geometrical figures are said to be congruent if they have exactly the same shape and size.

We used to adopt the method of super imposition to explain the idea of congruence of segments and angles. However the method is applicable in general to any two figures of the same type. But the method is not so convenient to use. It is indeed inpracticable. The reason is that 1) We cannot super impose a point on an other point when we take out a point from one plane to another point, it is no more that point. Then again what is a triangle ? Certainly it is a set of points. Can we remove and super impose them on an other set of points ? Certainly not. Let us take two sticks of the length of 25 cms. and 25.07 cms. When we compare their lengths by super imposition, do they not seem to tally exactly. Our necked eyes can't distinguish the exact position of a point. As such logically the method of establishing congruence between two figures cannot be considered as a logical proof. In its place we have accepted some geometrical postulates and axioms to be true and need no proof. The right axioms and postulates at right points make the study of axiometric

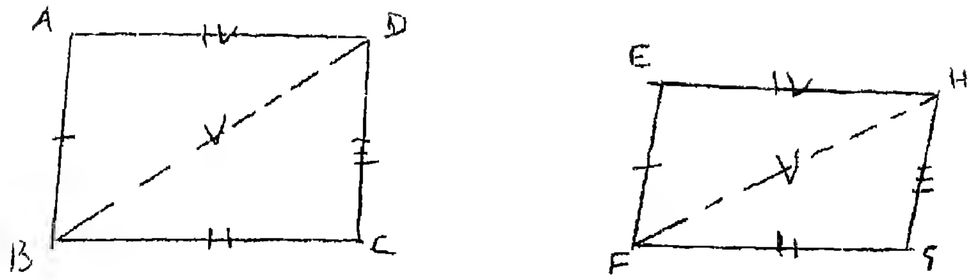
geometry prescribed by the B.S.E.Orissa more logical and consistent. That proves the excellence of the method adopted by the B.S.E.,Orissa. So our dear teachers can understand very well why the proof of theorem seven of the earlier publication of text books (S.A.S. theorem by Hall and Stevens) has been replaced by accepting the statement to be true as a postulate so also is the case as regards definitions of some geometrical concepts like point, line, plane etc. While trying to explain them, we often follow the cyclic logic. Hence they are not logical and from that point of view we have avoided definitions and accepted them as geometrically undefined terms.

With the help of S-A-S axiom, the congruence of all types of $\triangle$s have been proved under different sets of adequate data.

18.5 <u>Congruence of Qudrilaterals:</u>

For establishing congruence of two quadrilaterals we adopt the method of triangulation and proving the two triangles of one as congruent to two triangles of the other one to one, the quadrilaterals can be proved to be congruent. In proving two triangles congruent we compare three pairs of relevant parts. But in the case of proving two quadrilaterals congruent there is the need for 5 pairs of equal parts.

For example:



$\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{FG}$ and $\angle B \cong F$ are sufficient conditions for proving $\triangle ABC \cong \triangle EFG$ which implies $\overline{AC} \cong \overline{EG}$. Further two pairs of congruent parts like $\overline{AD} \cong \overline{EH}$ and $\overline{DC} \cong \overline{HG}$ will be adequate to prove that $\triangle ADC \cong \triangle EHG$ and hence we prove that quadrilateral ABCD $\cong$ quadrilateral EFGH.

18.6 Polygons: Regular and Convex:

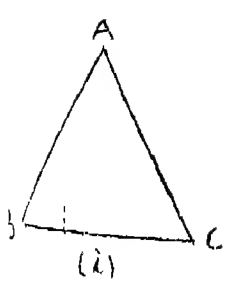
For $n \geq 3$ let $P_1, P_2, P_3, \ldots\ldots P_n$ be distinct points in a plane. If the n lines segments $\overline{P_1 P_2}$, $\overline{P_2 P_3}$, ... $\overline{P_{n-1}P_n}$, $\overline{P_n P_1}$ are such that :-

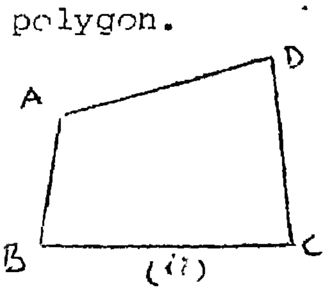
i) no two line segment intersect except at their end points,

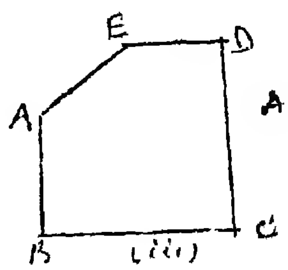
ii) no two line segments with a common end point are co-llinear, then

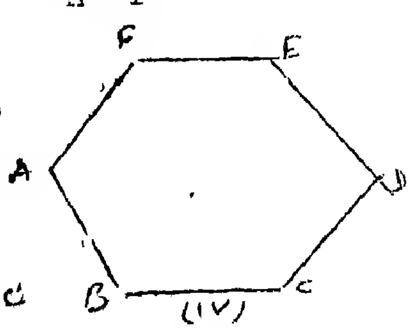
$\overline{P_1 P_2} \cup \overline{P_2 P_3} \cup \ldots\ldots \cup \overline{P_n P_1}$ is called

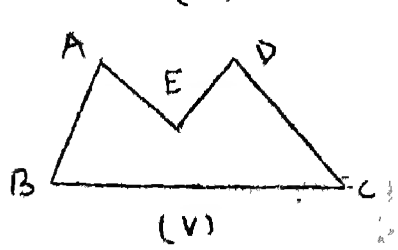
a polygon.

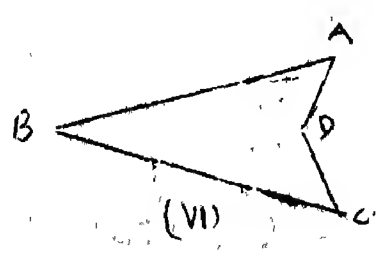












Convex Polygon:

A polygon $P_1 P_2 P_3 \ldots P_n$ is called a convex if for each side of the polygon the line containing that side has all the other vertices on the same side of it.

In this connection the polygons at the figures (v) and (vi) above are not convex.

Regular Polygons:

A polygon is said to be regular when all the sides are congruent with each other and all the angles are congruent with each other.

A quadrilateral polygon has four angles if it is convex. If it is not a convex quadrilateral, the sum of the measures of all its interior angles can't amount to 360°. So all we have discussed about is convex quadrilaterals.

A convex polygon is not a convex set. But it can be established that the interior of a convex polygon is a convex set.

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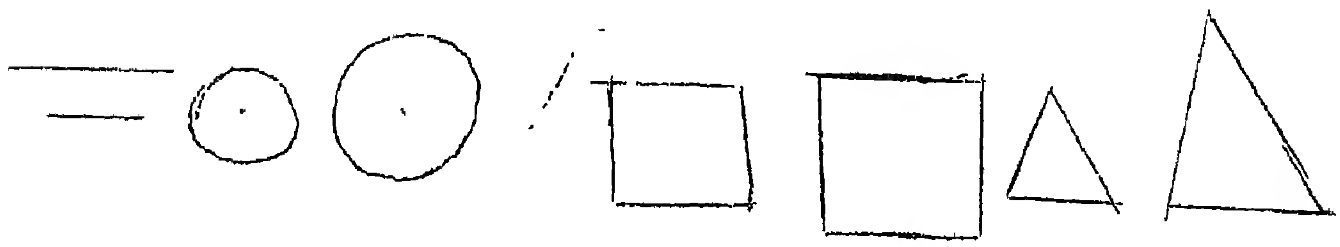
SIMILARITY

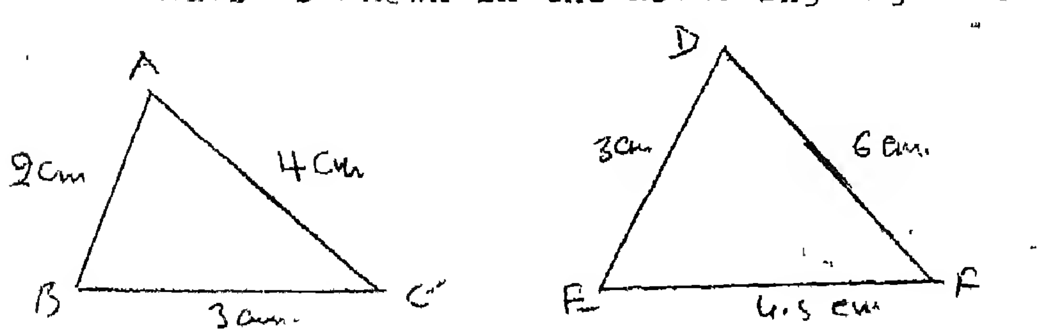
19.1 Proportionality:

Two geometrical figures are similar if they have exactly the same shape, but not necessarily of same size. For example, any two circles; any two squares, any two equilateral triangles and any two line segments are similar.



A student who has seen the map of India on the page of his book will not fail to recognise the map of India of much larger size hung against the wall. This is because of the fact that the two maps are similar in shape.

Let us take two $\triangle$s with lengths of their sides as shown in the following figures.



Here $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ and $\angle A \cong \angle D$, $\angle B \cong \angle E$

$\angle C \cong \angle F$ and under such situation. The triangles will have same shape for which we call them similar. $\triangle ABC$ is similar with $\triangle DEF$ and denote the idea by the symbol $\triangle ABC \sim \triangle DEF$.

Further propertionality of corresponding sides in $\triangle ABC$ and $\triangle DEF$ implies the congruency of the corresponding angles which can be proved. Thus it is a theorem. Here $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$. Conversely in two $\triangle$s ABC and DEF if $m\angle A = m\angle D$, $m\angle B = m\angle E$ and $m\angle C = m\angle F$ then

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

When the lengths of $BC = a$, $CA = b$, $AB = c$, $EF = d$, $FD = e$, $DE = f$ and

$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$ then a, b, c and d, e, f are two sequences of +ve numbers and are proportional. They can be symbolized as

$$a, b, c \sim d, e, f.$$

Properties of two sequences involving a simple proposition with a, b, c, d all > 0, if $\frac{a}{b} = \frac{c}{d}$ then the following properties follow:-

(i) $ad = bc$

(ii) $\frac{a+b}{b} = \frac{c+d}{d}$

(iii) $\frac{a-b}{b} = \frac{c-d}{d}$

(iv) $\frac{b}{a} = \frac{d}{c}$

(v) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

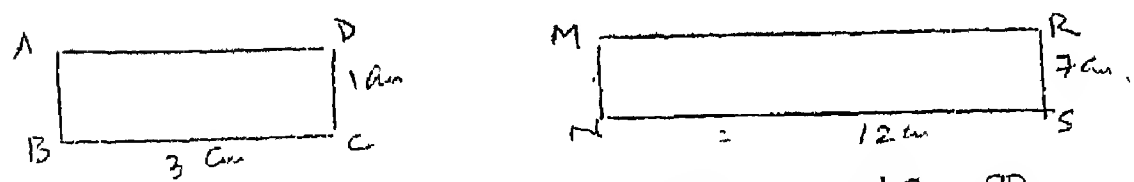
(vi) $\frac{a}{c} = \frac{b}{d}$

(vii) $\frac{a}{b} = \frac{c}{d} = \frac{a+b}{c+d} = \frac{a-b}{c-d} = \frac{b-a}{d-c}$

(viii) If a, b $\sim$ c, d then a, b and d, c are said to be inversely proportional.

If a, b, c are +ve numbers and $\frac{a}{b} = \frac{b}{c}$ then b is the geometric mean of a and c and we symbolise it as $b = \sqrt{ac}$

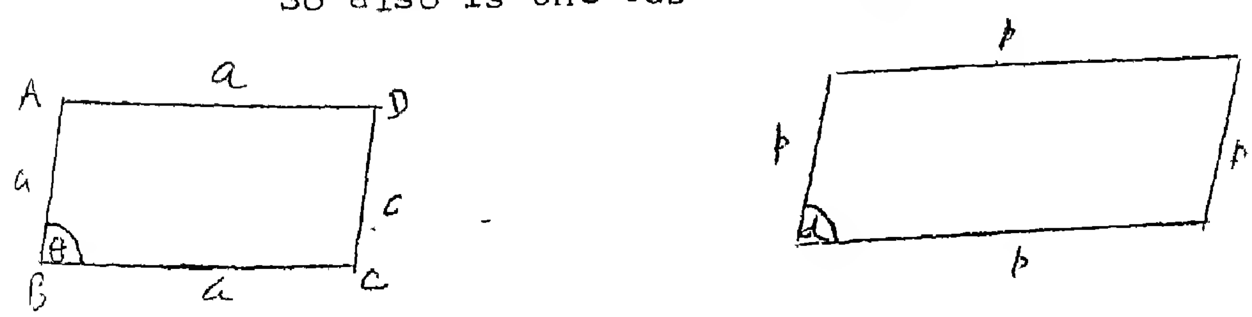
In case of two $\triangle$s, if the lengths of corresponding sides are proportional the corresponding angles are congruent and vice versa but in case of two rectangles or rhumbuses equality of angles need not imply necessarily proportionality of corresponding sides, nor proportionality of corresponding sides imply equality of angles.



Here neither $\frac{BC}{NS} = \frac{CD}{RS}$ nor $\frac{BC}{RS} = \frac{CD}{NS}$

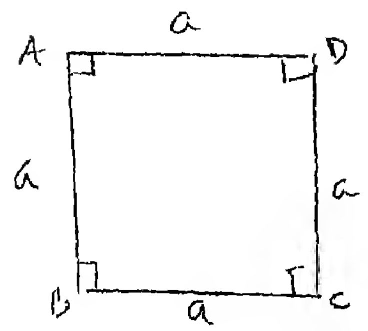
But $m\angle A = m\angle M$, $m\angle B = m\angle N$, $m\angle C = m\angle S$, $m\angle D = m\angle R = 90^{\circ}$

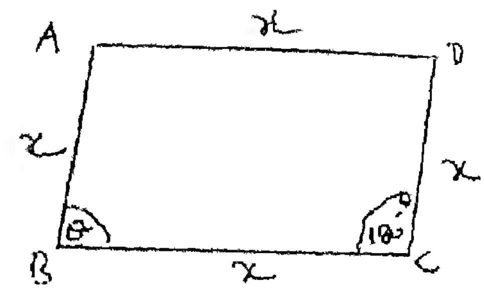
So also is the case in the case of two rhumbuses.



Here the lengths of the sides of the two rhumbuses are proportional but the corresponding angles are not congruent.

Then again let us consider about a square and a rhumbus.





Here the lengths of the sides are proportional but the corresponding angles are not congruent.

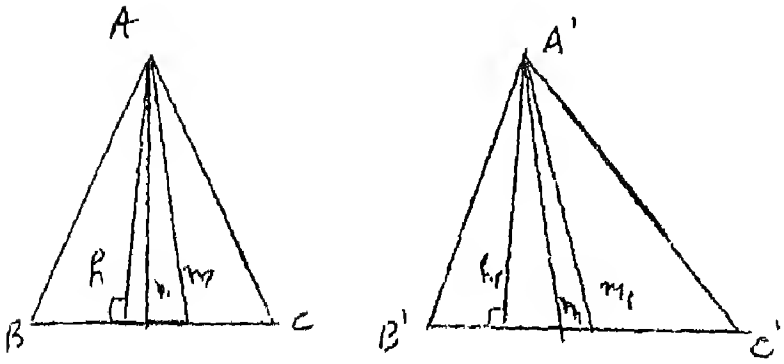
Thus we conclude that in case of two similar s proportionality of lengths of sides implies congruency of corresponding angles and vice versa but in case of quadrilaterals proportionality of lengths of sides may not imply the congruency of the corresponding angles and the vice versa.

19.2 Properties of Similarity:

1. $\triangle ABC \sim \triangle ABC$
2. $\triangle ABC \sim \triangle DEF \Rightarrow \triangle DEF \sim \triangle ABC$
3. $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle PQR$
   $\Rightarrow \triangle ABC \sim \triangle PQR.$
4. $\triangle ABC \cong \triangle DEF \Rightarrow \triangle ABC \sim \triangle DEF$
5. $\triangle ABC \sim \triangle DEF \Rightarrow \frac{a}{d} = \frac{a+b+c}{d+e+f}$
6. $\triangle ABC \sim \triangle DEF \Rightarrow \frac{ar\ \triangle ABC}{ar\ \triangle\ DEF} = \frac{a^2}{d^2} = \frac{b^2}{e^2} = \frac{c^2}{f^2}$

   if $A \leftrightarrow D$, $B \leftrightarrow E$ and $C \leftrightarrow F$.
7. If two triangles are similar, the ratio of the lengths of any two corresponding sides is proportional to the ratio of their

corresponding heights or median lengths or angle bisector lengths.



$\triangle ABC \sim \triangle DEF$

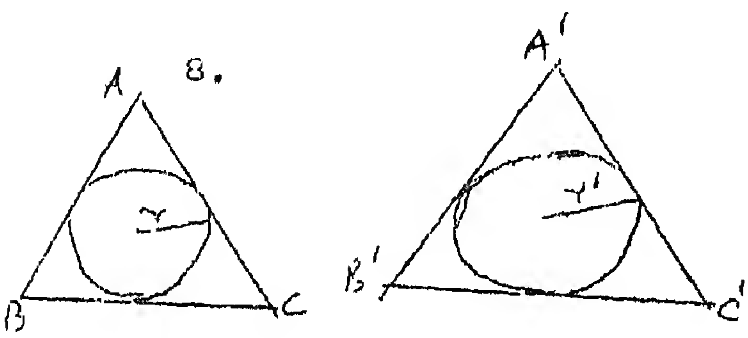
$A \longleftrightarrow A'$

$B \longleftrightarrow B'$

$C \longleftrightarrow C'$

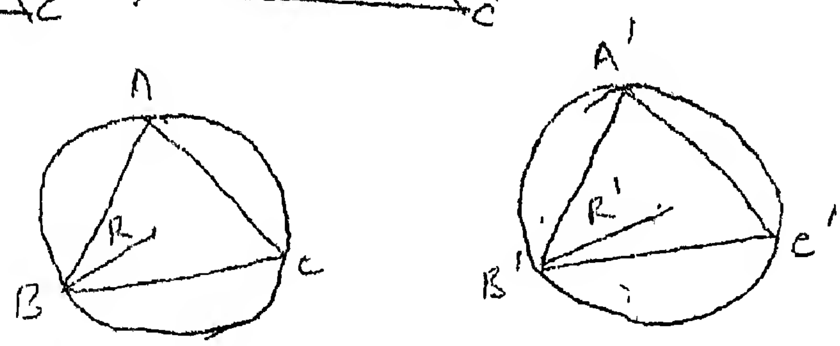
In the above case $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1} = \frac{h}{h_1} = \frac{m}{m_1} = \frac{n}{n_1}$

8.



In this case if $\triangle ABC \sim \triangle A'B'C'$ then $\frac{a}{a_1} = \frac{r}{r'}$

9.

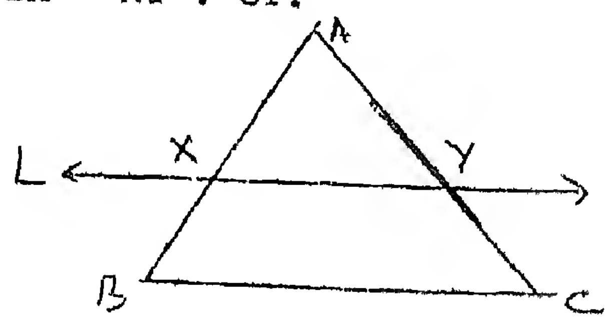


If $\triangle ABC \sim \triangle DEF$, then $\frac{a}{a'} = \frac{R}{R'}$ provided $A \longleftrightarrow A'$, $B \longleftrightarrow B'$ and $C \longleftrightarrow C'$.

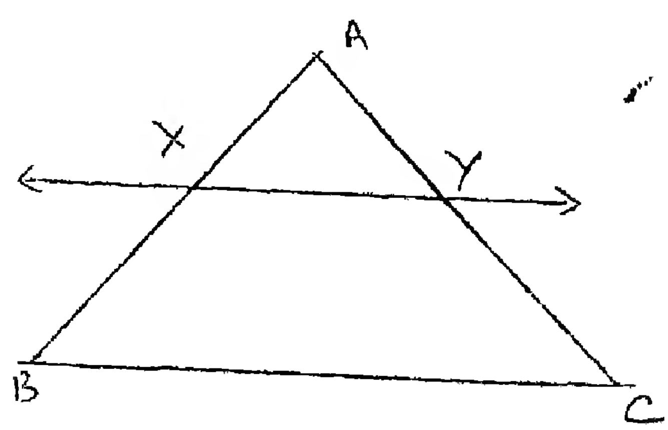
While taking into consideration the general enunciations of the theorems 1 and 2 of the Geometry and its application, P-II of the B,S,E, Orissa $X \neq Y$ is very much vital since this excludes the possibility of L passing through A. Formerly this restriction was not laid out. If student could assume that proposition, the theorems could not be proved.

Further more the question of the line L intersecting $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ at X and Y (X $\neq$ Y) either internally or externally and taking rigour of proving them doesn't arise now as it is provided algebrically by taking the help of the properties of ratio and proportion. Thus teaching of Geometry has been made easy.

Enunciation of theorem 1:- When a line L parallel to $\overline{BC}$ side of $\triangle$ ABC meets $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ at X and Y (X $\neq$ Y) respectively, then AX : BX = AY : CY.



Enunciation of the theorem 2:- If a line L interacts the sides $\overline{AB}$ and $\overline{AC}$ of the $\triangle$ABC at X and Y (X $\neq$ Y) respectively such that AX : BX = AY : CY, then the line L and $\overline{BC}$ are parallel.



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STATISTICS

INTRODUCTORY STATISTICS

20.1 Historical Background:-

The word "Statistics" has been derived from the Latin word "Status" or the Italian word "Statistika" or the German word "Statistik" which means a "Political State". In ancient time the government used to collect information regarding population or wealth of the country with a view to assessing the man-power of the State or to levy taxes. However statistics has been differently defined by different authors from time to time. The field of utility of statistics has been considerably widened in modern times statistics has been defined in two ways namely (i) Numerical data and (ii) Statistical method or Scientific method. By numerical data they mean numerical statement of facts having some characteristics and by statistical method they mean complete body of the principles and techniques used in collecting and analysing such data. However the definition given by Croxton and Cowden seems to be comprehensive which states "Statistics is the science which deals with the collection, analysis and interpretations of numerical data".

Administrative statistics existed in India some 2000 years ago. Kautilya's Arthasastra reveals that even before 300 B.C. there was a good system of collection of "Vital Statistics". During Emperor Akbar's regime (1556 - 1605 A.D.) Raja Todarmal,

the then Revenue Minister maintained records of land and revenue statistics. In Aina e Akbari written by Abulfazal there are detailed accounts of administrative and statistical survey of Akbar's reign.

By the end of 18th century, in order to ascertain the relative strength of the German States a system of official statistics originated particularly in the fields of population, industry and agricultural production. In England statistics was the outcome of Nepolienic wars which urged upon the government to immediately collect data to assess the collection of revenue to meet the war expenditure and to levy taxes. Captain John Grant of London is called the father of "Vital Statistics" as he was the first man to study the statistics of birth and death. In this regard Casper Newman, Sir William Petty, James Doson etc. did pioneering work which led to the idea of "Life Insurance" and the first Life Insurance Institution was founded in London in 1698.

Modern statistical theories are much to the mathematicians and gamblers of France, Germany and England with the introduction of "The Theory of Probability" and "The Theory of Games and Chances". For the development of statistics the works of the French mathematician Pascal, James Bernouli, De-Moivre, Gauss have contributed a lot. Modern Statisticians like Fransis Gallon, Karl Pearson, W.S. Gosset experimented with different statistical ideas.

Sir Ronald A. Fisher (1890 - 1962) is rightly called "the father of Statistics" as he applied statistics to diversified fields of human knowledge like genetics, education, agriculture, industry, medicine etc. It was he who placed statistics as a most reliable tool for the execution of fundamental research and for the development of all the disciplines of knowledge as a whole.

2.2 Definition of Statistics:

"Statistics may be defined as the Science of collection, presentation, analysis and interpretation of the numerical data". The following four important stages constitute a statistical investigation such as:

(i) Collection of data

(ii) Presentation of the data collected

(iii) Analysis of the data

(iv) Interpretation and conclusion reached at.

One may further consider it pertinent to incorporate a new stage in between stages (i) and (ii) namely proper organisation of the data.

Collection of data: In any statistical activity collection of data is the first step. The objective of statistical investigation determines what sort of data is to be collected from where and in which way. So there should be proper planning for the collection of the required data. The methods adopted should be reliable. Incorrect and faulty data ultimately result in erroneous conclusions.

20.3 Organisation of data:

The collected data is to be properly edited, classified and tabulated. If the investigator considers that some collected daa aren't worth using he may drop them and retain the rest for classigication basing on particular traits. Thereafter he may put them in tabular forms.

Presentation:

Orderly presentation of statistical data facilitates easy analysis. In two different ways data can be presented in the form of

(i) diagrams,
(ii) graphs,
(iii) tables
(iv) Frequency tables.

Analysis :

The stage that follows the above steps is analysis. The investigator carefully selects his method of analysing the data duly presented earlier in most of the cases in tabular form. Method of analysis may include methods like measure of central tendency, correlation or variation etc.

20.4 Interpretation:

The interpretation part of statistical investigation is the most important part as correct and intelligent interpretation leads to correct and reliable conclusion so that correct decision for

future is possible. Improper interpretation makes the whole exercise futile and the very purpose of investigation is bound to be defeated . This is the end product of statistical investigation.

20.5 STATISTICAL DATA

A set of numerical facts collected with a definite purpose in view is called statistical data or simply data. Depending on the source of collection they may be broadly divided into 2 categories:

(i) Primary data
(ii) Secondary data

Primary data are the one which are collected by the investigator through direct observation whereas secondary data are collected from already collected and stored pools of other agencies like published or unpublished works of others.

A choice between primary and secondary data depends on the following considerations:

(i) Nature and scope of investigation
(ii) Financial resources available
(iii) Period of time available.
(iv) Site of the collecting agency.

Primary data may be collected by the following methods:

(i) Through personal interview
(ii) Through indirect oral/written interviews
(iii) Through correspondences.

20.6 Raw data: The data collected at the initial stage in the original form is called raw data.

Example - 1: Ages (in years) of 20 students of a village selected at random:

13, 17, 11, 12, 13, 15, 12, 13, 12, 15, 11, 12, 16, 18, 14, 11, 10, 14, 15, 14 .

20.7 Presentation of data:

The raw data may be arranged and presented in different ways.

a) When the data is arranged in an ascending or descending order they form a line. This linear arrangement is called an array.

Example: The ages of the above 20 students may be arranged as

10, 11, 11, 11, 12, 12, 12, 12, 13, 13, 13, 14, 14, 14, 15, 15, 15, 16, 17, 18 or in the reverse order.

b) The same data may be arranged in a tabular form as in the following table:

Age in years	10	11	12	13	14	15	16	17	18
No. of students	1	3	4	3	3	3	1	1	1

Here we needn't write all the individual scores but we write the observed scores and the corresponding frequencies. Here frequency means the number of times by which a particular score occurs . For example in the above table the frequency of 12 is 4.

Example - 2: In a survey of 40 families in a village the number of children per family was recorded and the following data obtained. Present the data in the form of discrete frequency distribution:

1	0	2	3	3	2	5	0	2	4
4	2	4	2	3	2	7	3	1	4
5	3	4	3	8	5	2	5	6	11
2	12	9	7	4	6	7	2	10	1

From the above raw data it is clear that the number of children varies from 0 to 12 . So counting the repititions or occurrances of a particular score we frame the following frequency table:

No. of children (x)	Tallies	Frequency (f)
0	11	02
1	111	03
2	~~1111~~ 1111	09
3	~~1111~~ 1	06
4	~~1111~~ 1	06
5	1111	04
6	11	02
7	111	03
8	1	01
9	1	01
10	1	01
11	1	01
12	1	01
	Total =	40

20.8 Grouped data:

Suppose there are 30 students in a class and in a test out of 50 marks they obtain marks as follows:

25	20	16	18	30	35	40	16	20	18
25	30	46	40	30	5	10	25	18	20
30	25	44	28	35	30	25	25	20	30

As before the above raw data can be arranged in the following frequency distribution table:

Scores (x)	Number of students (frequency)
5	1
10	1
16	2
18	3
20	4
25	6
28	1
30	6
35	2
40	2
44	1
46	1
Total:-	30

We can also further condense or summerise the data into classes as follows:

Scores	No.of students(frequency)
1 - 5	1
6 - 10	1
11 - 15	0

Scores	No. of students (frequency)
16 - 20	9
21 - 25	6
26 - 30	7
31 - 35	2
35 - 40	2
41 - 45	1
46 - 50	1
Total =	30

This is called the frequency distribution table for grouped data. This type of frequency distribution is a grouped frequency distribution.

The above table doesn't take into account the score of each student. It rather classifies them into a series of groups. The scores are tabulated in groups or class intervals such as 1 - 5, 6 - 10, 11 - 15 etc. In 6 - 10 class interval 6 is the lower limit and 10 is the upper limit. The width of the class interval is 5.

The same data can also be presented in a different way as follows:

Scores	No. of students
0 - 5	0
5 - 10	1
10 - 15	1
15 - 20	5
20 - 25	4
25 - 30	7
30 - 35	6
35 - 40	2
40 - 45	3
45 - 50	1
Total:-	30

The two types of classifications discussed above are called inclusive Classification and exclusive classification . Classes of the type 0 - 5, , 5 - 10, 10 - 15 etc. are of exclusive type where the common point of the two classes is included in the higher class. Such classes are otherwise termed as continuous classes . Classes of the type 1 - 5, 6 - 10, 11 - 15 etc. are called inclusive type or discontinuous type. Were both the upper and lower limits of the class are included in the class interval itself.

In statistics we always deal with continuous class intervals, while discussing inclusive classes the question of actual upper and lower limits play a vital role. The actual upper limit is the mean of the upper limit of the lower class and the lower limit of the upper class. This mean is known as the adjusting factor. For example in case of the classes of the type 1 - 5, 6 - 10, 11 - 15 etc. the actual lower limit of the class 6 - 10 is $\frac{5 + 6}{2} = 5.5$ and the actual upper limit is $\frac{10 + 11}{2} = 10.5$. Thus to find the actual limits in the case of such inclusive classes 0.5 is added to the upper limit of the class and 0.5 is substracted from the lower limit of the class. Thus with the use of the adjusting factor the above inclusive classes become 0.5 - 5.5, 5.5 - 10.5, 10.5 - 15.5 and so on by which the classes become continuous. Similarly in the case of classes 0.5 - 5.5, 5.6 - 10.6, 10.7 - 15.7 etc. the adjusting factor will be 0.05.

The purpose of tabulation and classification is not the end of statistical procedure rather the first step to find the most representative scores which speak about the nature of distribution and is called the "Measure of central tendency".

MEASURES OF CENTRAL TENDENCY

21.1 Introduction:

A modern society is essentially information oriented. For all important human activities we need information in the form of numerical figures called statistical data. But when the volume of the data is large, we may not be able to draw any meaningful conclusion out of them. Thus the primary purpose of the statistical analysis i.e. to condense or summarise the raw data as far as possible without losing any information of interest. We have learnt how the data can be summarised to some extent by presenting them in the form of a frequency table. Another important and easily understandable method of presenting the data is the use of graphs e.g. histogram, frequency polygon, O-gives etc. Although frequency distribution and graphs serve useful purposes, they only focus on certain features of the data which describe their nature in a general way. When the data is collected with respect to a variable, it will be desirable to summarise them by means of some useful measures called descriptive measures. This is possible by calculating a single value called the central value of the variable

under study to represent or summarise the whole set of raw data. Such a representative score around which the other values of the variable cluster is called an average or a measure of location or a measure of central tendency. For example, if we study the data relating to the marks in Mathematics secured by the students in a group . We generally observe a tendency in the data to cluster around a certain value called the Average mark. So to compare the performances of the students in Mathematics between two different groups we shall compare only their average marks.

The commonly used measures of central tendency we shall study here are the Arithmatic Mean (A.M.) or simply mean, the Median and the Mode. Besides these three there are also other measures such as Harmonic Mean (H.M.) and Geometric Mean (G.M.).

The following are the simple characteristics to be satisfied by a good measures of central tendency.

1) It should be rigidly and clearly defined.
2) It should be easy to understand,
3) It should be easy to calculate,
4) It should be based on all the observations of the distribution.
5) It should not be affected much by the extreme values (i.e. the smallest and the largest values).

21.2 ARITHMATIC MEAN

21.2 The Arithmatic Mean of a set of observations is their Sum divided by the number of observations. For Example: the arithmatic mean of 5, 7, 9,, 10, 14, is

$$\frac{5+7+9+10+14}{5} = \frac{45}{5} = 9$$

In general if $x_1, x_2, x_3, x_4, \ldots\ldots, x_n$, are the n observation (Scores) of a distribution

$$\text{Mean } (m) = \frac{x_1 + x_2 + x_3 + x_4 + \ldots\ldots + x_n}{n} = \frac{1}{n} \sum_{i=1} x_i$$

$$\text{or} = \left(\frac{1}{n} \sum x\right)$$

Where M = Arithmatic Mean and the Greek letters $\sum$ (Sigma) means "Summation of ".

Example-1 The heights in centimeters of 10 players are 140, 150, 154, 148, 153, 145, 147, 157, 151 and 148.

Find the mean height of the players.

Solution: Here $n = 10$, $\sum x$ = Sum of the heights

= 1493 cm.

Hence Arithmatic Mean = $M = \frac{\sum x}{n}$

$= \frac{1493}{10} = 149.3$ cm.

21.3 Arithmatic Mean of a frequency distribution

If $x_1, x_2, x_3, \ldots\ldots, x_n$, are the different values with corresponding frequencies $f_1, f_2, f_3 \ldots\ldots f_n$,

$$\text{Arithmetic Mean} = M = \frac{f_1\,x_1 + f_2\,x_2 + f_3\,x_3 + \ldots f_n\,x_n}{f_1 + f_2 + f_3 + \ldots + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_i\,x_i}{\sum_{i=1}^{n} f_i} = \frac{1}{N} \sum_{i=1}^{n} f_i\,x_i \quad \ldots\ldots\ldots \text{II}$$

where $\sum_{i=1}^{n} f_i = N$ = Total frequency

Example-2: Find the mean of the following data:

x :	19	21	23	25	27	29	31
f :	13	15	16	18	16	15	13

Solution: We now arrange the data as given below:

x	f	fx
19	13	247
21	15	315
23	16	368
25	18	450
27	16	432
29	15	435
31	13	403
	$\Sigma f = 106$	$\Sigma fx = 2650$

$$\text{Arithmetic Mean} = M = \frac{\Sigma fx}{\Sigma f} = \frac{2650}{106} = 25$$

21.4 Arithmetic Mean of a grouped frequency distribution:

Consider the following frequency distribution.

Marks (x)	No. of students (f)
0 - 10	6
10 - 20	5
20 - 30	8
30 - 40	15
40 - 50	7
50 - 60	6
60 - 70	3

From the above table we see that the marks secured by individual students lose their identity assuming that all students in a class have marks equal to its mid point (or class mark) x_i of that class when $i = 1, 2, \ldots\ldots, 7$,

So we can compare the mean, M, by using our previous formula II for which the calculations are given below.

Marks (Class interval)	Mid point (x)	f	fx
0 - 10	5	6	30
10 - 20	15	5	75
20 - 30	25	8	200
30 - 40	35	15	525
40 - 50	45	7	315
50 - 60	55	6	330
60 - 70	65	3	195
		$\Sigma f = 50$	$\Sigma fx = 1670$

$$\text{Arithmetic Mean} = M = \frac{\Sigma fx}{\Sigma f} = \frac{1670}{50} = 33.4 \text{ marks}$$

We can now describe the procedure of computing the mean for a grouped frequency distribution as follows:

When the raw data is presented in the form of a grouped frequency distribution it is assumed that all the values falling in to a particular class interval are considered to be concentrated at the mid-point of a class obtained by $\frac{l_1 + l_2}{2}$ where l_1 and l_2 are the lower and upper limits of the class respectively.

These mid-points will be denoted by $x_1, x_2, x_3, \ldots, x_n$ if these are 'n' classes in the frequency distribution.

To find A. M we then use formula II i.e. we multiply the mid-point by the corresponding frequency, obtain the sum of the frequencies.

The mean is obtained by

$$M = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{1}{N} \sum_{i=1}^{n} f_i x_i$$

Where n = number of classes

x_i = the mid point of the i th class

f_i = the frequency of the 'i' th class

Example-3 The table below gives the expenditure in rupees on water consumption of 70 houses in a locality. Find the mean expenditure per house.

Expenditure on water (in Rs.)	No. of houses
15 - 20	7
20 - 25	5
25 - 30	7
30 - 35	8
35 - 40	9
40 - 45	11
45 - 50	7
50 - 55	5
55 - 60	4
60 - 65	4
65 - 70	3
Total:-	70

Solution: The work table for computing mean is as follows:

Expenditure on water	Mid-point x	Frequency f	xf
15 - 20	17.5	7	122.5
20 - 25	22.5	5	112.5
25 - 30	27.5	7	192.5
30 - 35	32.5	8	260.0
35 - 40	37.5	9	337.5
40 - 45	42.5	11	467.5
45 - 50	47.5	7	332.5
50 - 55	52.5	5	262.5
55 - 60	57.5	4	230.0
60 - 65	62.5	4	250.0
65 - 70	67.5	3	202.5
		$N = \sum f_i = 70$	$\sum fx = 2770.0$

Here $n = 11$, $N = \sum f = 70$, $\sum fx = 2770.0$

So the mean expenditure per house

$$= M = \frac{\sum fx}{N} = \frac{2770}{70} = \text{Rs. } 39.67$$

21.5 Short-cut method for computing mean:-

If the values of x (i.e. the mid values) and the corresponding frequencies are large the computation of mean becomes lengthy and tedious. In such a case the computation can be simplified by using a "short-cut method".

In this method we first choose an arbitrary constant 'a' (also called the Assumed Mean or origin) generally some where in the middle of all x - values. The reduced value ($x_i - a$) is called

the deviation of x_i from the asumed mean "a". These deviations are then divided by another constant 'c' which is generally taken to be the length of the class-interval. However, when there is no class-interval 'c' is taken to be 1.

$$\text{Let} \quad d_i = \frac{x_i - a}{c}$$

$$\text{or} \quad x_i = a + cd_i$$

We know that $M = \frac{1}{N} \sum_{i=1} fixi$

$$= \frac{1}{N} \sum_{i=1}^{n} fi\,(a + c\,d\,i)$$

$$= \frac{1}{N} \left\{ \sum_{i=1}^{n} fi\,a + \sum_{i=1}^{n} c\,fi\,di \right\}$$

$$= \frac{a}{N} \sum_{i=1}^{n} fi + \frac{C}{N} \sum_{i=1}^{n} fidi$$

$$M = \frac{a}{N} \times N + \frac{C}{N} \sum_{i=1}^{n} fidi \quad \left(N = \sum_{i=1}^{n} fi \right)$$

$$= a + \frac{C}{N} \sum_{i=1}^{n} fidi \quad \ldots\ldots\ldots \text{III}$$

<u>Example - 4</u> The heights (in cms.) of 12 students are given below. Calculate the mean by short cut method.

Height (Cm)	69	70	71	72	73
No. of students	4	2	3	2	1

Solution: Taking $a = 71$, $c = 1$ we have the following table.

Height x	No. of students f	$d = x - 71$	f d
69	4	-2	-8
70	2	-1	-2
71	3	0	0
72	2	1	2
73	1	2	2

$N = \sum f = 12$ $\qquad \sum fd = -6$

Now $M = a + \frac{1}{N} \sum fd$

$= 71 - \frac{6}{12} = 71 - 0.5 = 70.5$ cm.

Example-5 Find the mean age in year from the following frequency distribution by short-cut method.

Age(in year)	Frequency	Age(in years)	Frequency
15 - 19	3	35 - 39	5
20 - 24	13	40 - 44	4
25 - 29	21	45 - 49	2
30 - 34	15		

Solution: Here $C = 5$ and let us take $a = 32$

For computation of mean we have the following table.

Class interval	Frequency	Mid values of the class interval	$d = \frac{x - 32}{5}$	fd
15 - 19	3	17	- 3	- 9
20 - 24	13	22	- 2	- 26
25 - 29	21	27	- 1	- 21
30 - 34	15	32	0	0
35 - 39	5	37	+ 1	+ 5
40 - 44	4	42	+ 2	+ 8
45 - 49	2	47	+ 3	+ 6
	N = 63			$\sum fd = - 37$

So the mean age

$$= M = a + \frac{C}{N} \sum fd$$

$$= 32 + \frac{5 \times (-37)}{63} = 29.06 \text{ years}$$

21.6 <u>Merits and Demerits of Arithmetic Mean:</u>

<u>Merits:</u> (1) It is easy to calculate and easy to understand.

(2) It is rigidly defined.

(3) It is based on all observation.

<u>Demerits:</u> (1) It may be greatly affected by the extreme values.

(2) It is not a suitable average for a distribution with open end classes such as less than 10 or more than 70 etc. because in this case the midpoint of the class-interval can not be determined.

(3) It can neither be determined by inspection nor can it be located graphically.

(4) It can not be used while dealing with qualitative characteristics such as intelligence, honesty, kindness, beauty etc. which can not be measured quantitatively.

(5) It can not be determined if a single observation is lost or missing.

21.7 Weighted Arithmatic Mean

Let us consider the marks secured by a student in 4 different subjects.

Subject :	I	II	III	IV
Full Mark	50	75	100	100
Marks obtained	30	65	40	55

Here we can compute the mean = $\frac{30 + 65 + 40 + 55}{4}$

$$= \frac{190}{4} = 47.5$$

But this figure can not be a proper representative of the four marks . Because while calculating the mean we assume that all the subjects are of equal importance. We have not taken the full marks of a subject in to consideration. If we complete the mean by taking into account of the full mark, called weights (or importance), then it will be a proper representative of four marks such a mean is called a weighted mean. Thus our weighted mean in this case will be

$$M = \frac{50 \times 30 + 75 \times 65 + 100 \times 40 + 100 \times 55}{50 + 75 + 100 + 100}$$

$$= 50.38$$

Definition:- If x_1, x_2, x_3 x_n denote n values of a variable x, and w_1, w_2, w_3, w_n denote respectivel their weights then their weighted mean M_w is given by:;

$$M_w = \frac{\sum_{i=1} Wi \; xi}{\sum_{i=1} W\,i} \quad \ldots\ldots\ldots(iv)$$

As a special case the AM defined by formulla I can be considered as a weighted mean with the weight in each case being 1. Similarly the AM defined by formula II can be considered as a weighted Mean with weights being frequencies. The calculation of weighted mean is very useful for computing index numbers. Because the prices of the commodities consumed by a group of people are not equally important i.e. wheat, rice, lighting, and medicines are more important than cosmatics, cigarrettes, tea etc.

Example - 6 A candidate obtained 60, 75 and 85 marks respectively in three monthly examinations in Mathematics and 95 marks in the final examination. The three monthly examination are of equal weightage where as the final examination is weighted twice as much as a monthly examination . Find his mean marks in Mathematics.

Solution: Here $n = 4$, $x_1 = 60$, $x_2 = 75$, $x_3 = 85$ and $x_4 = 95$ $w_1 = w_2 = w_3 = 1$ $w_4 = 2$ then by formula

$$M_w = \frac{1 \times 60 + 1 \times 75 + 1 \times 85 + 2 \times 95}{1 + 1 + 1 + 2}$$

$$= \frac{410}{5} = 82$$

So Mean = 82 marks.

MEDIAN

22.1 Introduction:

Suppose we want to have an idea on the average height of students in a group. Here the computation of A.M. can not be done quickly, because for this we have to measure the height of each student. But if we arrange the students in a line with their height in an order (i.e. either in increasing order or in decreasing order) then the measurement of the height of the student standing in the centre can be taken as their average. This can be done quickly without taking measurement of individual height. Such a measure of central tendency is called the median.

Definition:- The median is that value of the variable which devides the distribution into two equal parts in such a manner that the number of observations below it is equal to the number of observation above it.

Thus if the given value of the variable x are arranged in an order, the middle most value in this arrangement will be the median of x. When the number of values n, is odd, the middle most value i.e. $\frac{n+1}{2}$ the value in the arrangement will be the median. On the otherhand if n is even there may be no unique median. Because any value in between $\frac{n}{2}$ th and $(\frac{n}{2}+1)$th values of x being regarded as the middle most value can be taken as a median. However for definitions, AM of $\frac{n}{2}$ th and $(\frac{n}{2}+1)$th values is accepted as the median of x. Then we have median $M = \frac{n+1}{2}$ th value, if n is odd and $= \frac{1}{2}\left[\frac{n}{2}\right.$ the value + $(\frac{n}{2}+1)$thof $\left.\text{value}\right]$ if n is even-V.

Let us consider the following examples.

(i) Let 60, 62, 69, 70, 68, 63, 64, 66, 68, 69, 70 be the heights (in Cm.) of 11 students.

The values arranged in increasing order are 60, 62, 63, 64, 66, 68, 68, 69, 69, 70,70.

Hence n is odd i.e. $n = 11$

The median $M_d = \frac{n+1}{2}$th value = 6th value

Hence $M_d = 68$

(ii) Let 55, 50, 56, 54, 51, 44, 47, 46 , 54 and 44 be the marks secured by 10 students in English. The values arranged in increasing order are 44, 44, 46, 47, 50, 51, 54, 54, 54,55, 56.

So the mean of $\frac{n}{2}$ th value i.e. 5th value and ($\frac{n}{2}$ + 1)th value i.e. 6th values will be the median . Hence $M_d = \frac{1}{2}(50 + 51) = 50{\cdot}5$

Median for a frequency distribution

Example-7 Calculate the median for the following distribution.

Weight (in Kg.)	26	27	28	29	30	31
No.of students	3	5	6	7	6	4

Solution: In this case the values are arranged in an order. So we can find out the median if we can only locate the middle most item (students). Consider the following cumulative frequency table.

Weight (in Kg.) x	No. of students f	Cumulative frequency. cf
26	3	3
27	5	8
28	6	14
29	7	21
30	6	27
31	4	31

$$N = \sum f = 31$$

Here: n = 31, which is odd. The position of median is $\left(\frac{n+1}{2}\right)$ th or $\left(\frac{31+1}{2}\right)$ th or 16th item (student).

So M_d = 16th value of the distribution .

But from the above c.f. table we see that the value of all the items from 15th to 21st is 29. So value of the 16th item is also 29. Median M_d = 29 Kg.

Example - 8: Find the median of the following item.

X :	6	7	8	9	10	11	12
f :	25	39	48	43	52	30	13

Solution: Forming the cumulative frequency table we get

x	f	Cumulative frequency cf
6	25	25
7	39	64
8	48	112
9	43	155
10	52	207
11	30	237
12	13	250

Here $n = \sum f = 250$, which is even

$\therefore$ Median is the average of $\frac{n}{2}$ th and $(\frac{n}{2} + 1)$ th values.

As such here the median is the mean of 125th and 126th values = 125.5 th value.

Therefore the median $M_d = 9$

Now we point out the following general procedure to find out median for a given frequency distribution . The steps are as follows:-

(i) Prepare the c.f. distribution table.

(ii) Find $\frac{N}{2}$

(iii) Find out c.f. just greater than $\frac{n}{2}$.

(iv) The corresponding value of the variable gives the median.

This procedure of computing median can be verified with the help of examples 7 and 8.

22.2 Median for a grouped frequency distribution

In a grouped frequency distribution the values of the variable x are divided into a suitable number of class intervals . So after forming the c.f. distribution table, We shall identify the class whose c.f. is just greater than $\frac{n}{2}$. This class contains the median value and is called the median class. The value of median is thus obtained by using the following formula.

$$M_d = l_1 + \frac{l_2 - l_1}{f} \times (\frac{N}{2} - C) \quad \ldots\ VI$$

Where l_1 and l_2 are respectively the lower and upper values of median class, f = frequency of median class and C = c.f. of the class preceeding the median class.

Example-9: Calculate the median for the following frequency distribution.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	2	2	7	13	15

Marks	50-60	60-70	70-80	80-90	90-100
Frequency	12	9	6	3	1

Solution: Consider the following c.f. table .

Marks (x)	Frequency (f)	Cumulative frequency (c.f.)
0 - 10	2	2
10 - 20	2	4
20 - 30	7	11
30 - 40	13	24
40 - 50	15	39
50 - 60	12	51
60 - 70	9	60
70 - 80	6	66
80 - 90	3	69
90 - 100	1	70

$$N = \sum f = 70$$

Hence $\frac{N}{2} = 35$ and c.f. just greater than 35 is 39.

So median class is 40 - 50.

Then $L_1 = 40$, $L_2 = 50$, $f = 15$ and $C = 24$

$$\text{Median} = M_d = L_1 + \frac{l_2 - l_1}{f}\left(\frac{N}{2} - C\right)$$

$$= 40 + \frac{50 - 40}{15}(35 - 24)$$

$$= 47.33$$

22.3 Location of Median by graphic method:-

The median can also be determined approximately from the ogine i.e. the cumulative frequency curve. Let us consider the following examples.

Example-10: For the given frequency distribution draw the ogive and hence find out median.

Value (x)	5	6	7	8	9	10	11	12	13	14
Frequency	6	8	8	11	22	36	59	29	21	3

Solution: Let us construct the following c.f. distribution table.

x	-	5	6	7	8	9	10	11	12	13	14
f	-	6	8	8	11	22	36	59	29	21	3
cf	-	6	14	22	33	55	91	150	179	200	203

Draw ogive by taking x-values along x - axis and corresponding cumulative frequency along Y-axis. Compute $\frac{N+1}{2}$ and locate the point on Y- axis i.e. $\frac{204}{2} = 102$. Draw a line parallel to X-axis at the point so that it meets the ogive at a point P. Draw a perpendicular on X-axis from the point P to meet the X-axis at M_d. The point M_d gives the value of the median.

22.4 Merits and de-merits of Median.

Merits: (i) Median is rigidly defined.

(ii) It is easy to understand and easy to calculate even o f for a non-mathemetical person.

(iii) Median can be computed for a distribution with open end class.

(iv) Unlike mean it is not affected by the extreme values.

(iv) It can sometimes be located by simple inspection and can also be computed graphically.

De-merits (i) It is not based on all observations.

(ii) In case of even number of observations median can not be calculated exactly.

(iii) Median is usually less stable than the mean.

Uses of Median:- Median is the only average to be used while dealing with qualitative characteristics which can not be measured quantitatively but can still be arranged in order i.e. to find out the average intelligence, average beauty, average honesty kindness etc. among a group of people.

....

M O D E

23.1 Introduction:

Consider the following statements:

i) Average height of an Indian male is 1.68 M

ii) Average expenditure of a student in a hostel is Rs.300/- per month.

iii) Average size of shoe sold in a market is size No.7 .

In these cases the average referred to is neither mean nor median, but the value of the variable x(i.e. height or expenditure or size) that occurs or repeats itself the greatest number of times. Such an average is called mode. For example, in the third statement we mean that there is maximum demand for shoe of size No.7 . Thus mode is 7.

Definition:- Mode is the value of the variable which occurs most frequently in a distribution and around which the other values of the distribution cluster densely.

Example - 10 If 7 men are receiving daily wages of Rs.60, 50, 70, 70, 80, 100, 70 find out the model wage.

After arranging the values in increasing order i.e. 50, 60, 70, 70, 70, 80, 100 it is clear that 70 occurs 3 times .

Hence Mode = Mo = Rs.70 per day.

If in a given series the occurances of different values are equal, we shall say that there is no mode. For example in case of a series like 3,5,7,3,8,5,3,7, there is no mode.

Computation of mode for a frequency distribution

In case of a frequency distribution, mode is the value of the variable corresponding to the maximum frequency.

Example - 11: Calculate mode

x :	1	2	3	4	5	6	7	8	9
f :	3	1	18	25	40	30	22	10	6

Here the maximum frequency is 40 and the corresponding value of x is 5. Hence Mode = Mo = 5

Example - 12: Find mode for the following distribution

No.of children (x)	0	1	2	3	4	5
No.of families (f)	10	21	55	42	55	15

Here the frequencies of 2 and 4 are the maximum i.e. 55.

Thus Mode = Mo = 2 and 4

For a grouped frequency distribution the class interval corresponding the maximum frequency is called the modal class. Then mode is calculated by writing the following formula.

$$Mo = l_1 + \frac{(l_2 - l_1)\ (f_1 - f_o)}{2f_1 - f_o - f_2} \qquad \text{VIII}$$

where l_1 and l_2 are respectively the lower and upper limits of the modal class, f_1 is the frequency of modal class, f_o is the frequency of the class preceding the modal class and f_2 is the frequency of the class succeeding the modal class.

Example - 13: Calculate the mode for the distribution of the weights of 150 students from the data given below.

Weight (in Kg)	Frequency
30 - 40	18
40 - 50	37
50 - 60	45
60 - 70	27
70 - 80	15
80 - 90	8

Here the maximum frequency is 45. So that the modal class is 50 - 60. Thus we have

$l_1 = 50,\ l_2 = 60,\ f_1 = 45,\ f_o = 37$ and $f_2 = 27$

$$\text{Hence, Mode} = M_o = l_1 + \frac{l_2 - l_1)\ (f_1 - f_o\ )}{2f_1 - f_o - f_2}$$

$$= 50 + \frac{(60 - 50)\ (45 - 37\ )}{2 \times 45 - 37 - 27}$$

$$= 50 + \frac{10 \times 8}{26}$$

$$= 50 + 3.077$$

$$= 53.077 \text{ Kg.}$$

23.2 Merits and Demerits of Mode:

Merits: i) Mode is easy to calculate and easy to understand.

ii) Mode is not affected by the extreme values

iii) Itc an be easily calculated in the case of open end classes.

De-merits: i) Mode is not clearly defined. Because when the maximum frequency is repeated we can locate more than one modal value.

ii) It is not based on all observation.

Uses of Mode:

Being the point of maximum frequency, mode is specially useful in finding the most popular size in studies relating to marketing, trade, business and industry. It is the only appropriate average to be used to find the ideal size e.g. in business forecasting in the manufacture of shoes or ready made garments, in sales, in production etc.

23.3 SELECTION OF AN AVERAGE

From the discussion of the merits and de-merits of the various measures of central tendency we see that no single average is suitable for all practical problems. Each average has its own merits and de-merits and consequently its own field of importance and utility. For example, AM is not suitable while dealing with frequency distribution with the terminal class intervals are open, "like less than 50" or "greater than 1000" one can use here either median or mode. In case of qualitative data which can not be measured quantitatively median is the only average to be used. Mode is particularly used in business.

Hence an average can not be used indiscriminately. For better statistical analysis a judicious selection of an average depends upon:

i) The nature and the availability of data.

ii) The nature of the variable studied

iii) The purpose of the study.

iv) The method of classification adopted.

However, since A.M. satisfies almost all the properties of a good average, is quite familiar to a layman and has very wide applications in the statistical theory at large, it may be regarded as the best of all averages.

INDEX NUMBER

24.1 Introduction:

All of us are well acquinted with the price of different commodities in our day to day life. The prices of all commodities change (i.e. increase or decrease) from time to time due to different factors like increase in population, variation in agricultural and industrial production, transportation facilities, family status and demands etc. So the required expenditure for a family changes according to the fluctuation in market price. In this situation the salaries or wages of workers or employees are need to be changed to maintain a balance with the change of market price. Keeping in mind the above change it is desirable to determine an indicator which will reflect on the change of living standard of a family (or a group of people) for a given year (called current year) compreed to any previous year (called base year). Such an indicator in this case is called a price index number.

For example - suppose that the rate of rice per Kg. in 1996 (as current year) and 1994 (as base year) are Rs.10.00 and Rs.8.00 respectively. Then the price index of rice for 1996 with respect to 1994 will be $\frac{10}{8} \times 100 = 125$. From the example it is clear that the price index of a perticular commodity in the current year is expressed as a percentage of all the price at base year.

In practice we deal with the prices of several commodities simultaneously. In this case the index number will be considered as a special type of

weighted mean calculated by expressing the total expenditure of the current year as a percentage of the total expenditure in the base year.

Besides the price index number, there are other two index numbers. They are the quantity index number and consumer price index number or cost of living index number.

24.2 Cost of living index number:-

Cost of living index number is a price index number with special reference to a class or a category of people in a society at different times or in different region.

Suppose we select a group or section of people and find out its monthly consumption on certain standard items during 1990 and 1996. We find the total expenditure on these items as per the existing rates in 1990 as well as 1996 assuming the quantities consumed in 1990 are also consumed in 1996. Then the cost of living index in 1996 with respect to base year 1990

$$= \frac{\text{total cost in 1996}}{\text{total cost in 1990}} \times 100$$

Thus in general the cost of living index =

$$\frac{\sum p_1 q_1}{\sum p_o q_o} \times 100 \qquad \text{(viii)}$$

Where p_o = Price of the commodities per unit in the base year.

q_o = quantity of the commodities in the base year.

P_1 = Price of the commodities per unit in the current year.

Example - 14: Calculate the cost living index number from the following data for 1995 taking 1990 as base year.

Commodities	Quantity used	Rate per unit in 1990	Rate per unit in 1995
Rice	40 Kg.	Rs.4.00	Rs.5.00
Oil	5 Kg.	Rs.25.00	Rs.30.00
Sugar	7 Kg.	Rs. 7.00	Rs. 9.00
Milk	15 Ltrs.	Rs. 4.00	Rs. 6.00
Meat	4 Kg.	Rs.60.00	Rs.70.00

Solution: Here the quantity of the items consumed in both the years are the same i.e. $q_1 = q_o = q$ (say)

We now consider the following table

Commodities	Quantity used	Rate per unit in base year 1990 P_0	Rate per unit in current yr. (P_1 = 1995)	$P_o \times q$	$P_1 \times q$
Rice	40 Kg.	Rs.4.00	Rs.5.00	Rs.160/-	Rs.200/-
Oil	5 Kg.	Rs.25.00	Rs.30.00	Rs.125/-	Rs.150/-
Sugar	7 Kg.	Rs.7.00	Rs. 9.00	Rs. 49/-	Rs. 63/-
Milk	15 ltr.	Rs.4.00	Rs.6.00	Rs. 60/-	Rs. 90/-
Meat	4 Kg.	Rs.60.00	Rs.70.00	Rs.240/-	Rs.280/-

$\sum P_o\, q$ = Rs.634.00 $\quad \sum P_i\, q$ = Rs.783.00

Here the cost of living index number

(using formula VIII)

$$= \frac{\sum P_1\, Q}{\sum P_o\, Q} \times 100$$

$$= \frac{783}{674} \times 100$$

$$= 1235$$

.....

COMPUTING

COMPUTER

"The basic controversy with Computers is that it's very hard to learn the Software and it's very Soft to view hard work.

25.1 What is a Computer ?

Literally Computer means a machine or an equipment which does calculations. But the Computer we use is an electronic device which can process, store, retrieve and transmit information which may be audio, visual or linguistic type. More elaborately a Computer is an electronic device capable of manipulations numbers and symbols, taking an input storing it, processing it and giving an output, all under the control of a set of instructions called a Programme. However a Computer performs only three basic functions internally:

Arithmetic calculations (i.e. additions, subtractions, multiplications and its raciprocal), comparing two data items and moving data items from one internal memory location to another.

25.2 Evolution of Computer

The evolution of Computer has passed through a number of stages before it reached the present state of development. Infact, the development of the first calculating device named ABCUS dates back to 3000 B.C. From ABACUS to the micro Computer, the calculating systems have undergone a tremendous changes. A brief account of the development is given below:

25.1 Early Calculating Devices

Abacus: The stone age man used pebbles for counting cattle. Latter on when man became more civilized, abacus came in use. Abacus seems to be the earlier calculating machine, which was developed by Chinese 3000 years ago. It consists of a frame with some bars fixed accross it. Each bar had some beads which could be moved along each bar. Every bead represented a position in the number system, i.e. units, tans, hundreds ..... etc.

Napier's Bones

John Napier, the Scottish Mathematician devised a set of rods for use in calculations involving multiplications. These rods were curved from bones and therefore called as Napier's Bones. Napier used 10 stripes of bones and divided each into 9 squares. Again each square was divided in to two parts. The lower half indicated units and the upper half indicated tens. He used the principles of multiples to fill the squares.

Oughtrad's Slide Rule

In 1620, William Oughtrad invented the slide - rule which is a calculation device that used the principle of logarithms, invented earlier by John Napier.

Pascal's Calculator

In 1642, the great French Mathematician and Philosopher Blaire Pascal devised a calculating machine which consists of gears, wheels and dials.

Each wheel had 10 segments like that of milometers, when one wheel completed a rotation, the next wheel moved by one segment. With this calculator all arithmetical calculations by dialing these series of wheels bearing the numbers 0 to 9 around their circumference.

Jacquard's Loom

A French Weaver, Joseph Jacquard, used punched cards to determine the threads to be selected in weaving patterns automatically. A role on the Card permitted a hooked wire containing a thread to enter the pattern which is the absence of a hole would correspondingly prevent.

Babbage's Difference Engine and Analytical Engine

Charles Babbage (1792 - 1871) Professor of Mathematics at Cambridge University, a genius in history of Computing, made a machine called difference engine which could evaluate accurately algebraic expressions and mathematical tables correct up to 20 decimal places.

Latter he proposed an Analytical Machine which was an automatic computing machine designed to do additions at the rate of 60 per minute and had a memory also. Babbage could have designed better machine during his life time itself but the contemporary technology could not assist and match his genius. He determined to add to the arithmatic capability of ~~the then~~ existing calculators, the ability to store intermediate

results for subsequent calculations. This engine eliminates re-sortering, as it consists of a memory and more important was the fact that data could be entered along with the sequence of operations to be performed on data. He used punched cards to enter data. His idea had all the basic components of today's modern Computer. Therefore, he is called the Father of Modern Computer.

Herman Hollerith - Punched Cards

Dr.Herman Hollerith of U.S.A. was the next contributor to development of Computers. He was working in the U.S.A. Census office when data was being compiled and analysed manually. In order to overcome this tedious job he invented a machine which used punched cards to store and tabulate Census informations. This machine could sense the punched holes, recognise the number , and make the required calculations.

25.2.2 Middle Age Computers

Electrical Machines

During the late thirtees and early forties of this Century, various types of Computers with very unusual names were build in Germany, U.K. and in U.S.A. The progress in Computer technology was accelerated during the Second World War. Computers were used in military operations like breaking codes and for designing air Crafts. Conrad Zuse a young German engineer started in 1936 to make a model Mechanical Computer Z, which had a Key board

for putting the numbers into machine. It used the binary system which enabled him to take advance of Boole's system of logic. In Z_2 Conrad Zuse replaced the slow mechanical switches with electrical relays.

IBM Mark 1

In 1937, Harvard Professor Howard Aiken set out to build an automatic calculating machine to combine electrical and mechanical technology with Hollerith's punched Card technique. With the help of the students and Engineers the project was completed in 1944. The completed device was known as the Mark - 1 - Digital Computer. The mark was an electromechanical Computer of length 15 mt and weighed nearly 1500 tonnes and computed 20 multiplications in one second.

25.2.3 Modern age Computers

ABC (Atanassoff - Berry - Computer)

During 1937-38 Dr.John Vincene Atanassoff, a Professor of Physics and Mathematics teamed up with Clifford Berry, his graduate assistance and began work on an electronic Computer and finished a working prototype in 1942. This Computer known as ABC (Atanassoff - Berry Computer) was later accepted as the first Computer.

ENIAC

Another electronic based machine was made in 1946 byJ.P. Eckert and J.W.Mauchly at the

University of Pennysylvania of U.S.A. and was called Electronic Numerical Integrator and Calculator (ENIAC). It was a very fast machine as compared to its ancestors which could perform 5000 additions or 3500 multiplications in one second. It was completed in 1946.

First Generation Computers (1942 - 1955)

The first generation Computers were voluminous computers. These computers used electronic valves like the ones used in radios. ENIAC and various other computers of large main-frame type which fall in first generation category are UNIVAC - 1 , IBM 701, IBM - 650. The Computers in the first genration were found to be fast, accurate and untiring processors of mountains of paper.

Second Generation Computers (1955 - 1964)

The invention of transistor in 1948 led to the development of Second generation computers. Transistors replaced the valves completely as they were far more superior in performance on account of their miniature size, smaller power consumption and heat production rate. The use of transistors in Computer reduced (i) Size (ii) Manufacturing and running costs and improved (iii) reliability and processing power.

Some Second generation computers are IBM ' ,
1620, IBM 1401, UNIVAC 1108 etc. The computers of the Second generation which began to appear in 1959 were made smaller and faster and had greater computing activity.

Third Generation Computers (1964 - 1975)

In 1964, the Third generation computers were introduced. These had integrated transistor circuits (I.C.) having higher speed, larger storage capacities and lower prices. These computers were called minicomputers. These computers used integrated circuits (I.C.) built on wafer - thin slices of extremely purified silican crystal called chip. The manufacture of minicomputers of third generation began after development of large scale integration (LSI) and very large scale integration (VLSI) and microprocessor chips. Availability of LSI and VLSI led to the production of variety of mini computers some of which can perform main frame computers in the 1960s. These are small - size, low cost, large memory ultrafast computers which are proliferal at a surprising rate.

Fourth Generation Computers (1975 onwards)

The Computers built in 1960s were mainframes designed to provide at a central site, all the processing power needed by an organisation. This approach served the needs of some organisations. But others were unable to afford large systems. This led to the requirement of low-cost minimal computers to fill the gaps by the bigger, faster, centralised approach. This led to the innovation of mini computers and micro computers.

ICs which has the entire computer circuits on a single silican chip are called microprocessors. The computers using these chips are called micro processors. The computers using these chips are called micro computers. These are infact, the scaled down versions of mini computers.

Fifth Generation Computer (Yetlo come)

Currently, a high stakes competition between American and Japanese computer manufactures exist. The prize in this multibillion dollar contest as the "Thinking Computer". Thus in a computer which will be able to handle facts and ideas, make inferences and deductions and answer questions and problems in the smallest fraction of a second. The "Thinking Computers" make a fifth generation of Computers.

Scientists are now at work on the fifth generation computers - a promise but not a reality. They aim to bring as machines with genuine 1.Q, the ability to reason logically, and with real knowledge of the world. Thus unlike the last four generations which naturally followed its predecessor, the fifth generation, if succeeds, will be totally different, totally novel, totally new.

25.3 CLASSIFICATION OF COMPUTERS

Computer systems are classified by their C.P.U sizes, number of online terminals, other available I/Q devices in to following categories.

25.3 CLASSIFICATION OF COMPUTERS

Computer systems are classified by their C.P.U. sizes, number of online terminals maximum disk storage capacity, and all other available I/O devices in to following categories.

MICROCOMPUTERS

Microcomputer is called micro for two reasons: One, because it is miniature in size and another because it uses micro-processor as its Central Processing Unit. Microprocessor is actually the heart cum mind of a computer. It is contained in a single chip made of sillicon. The PCS (Personal Computers) commonly used are examples of microcomputers. PCs are of three sizes: (i) Disk tops (ii) Note books and (iii) Palm top.

Desk top PCs have a large screen and a full size key board. Note book PCs are about the size of a note book and weigh less than 4 Kgs. but they have all features of a desk top. Palm top PCs are smallest in size and weigh less than 500 gms.

Mini Computers:

A mini computer is larger in size than a PC. It has higher storage capacity, works faster than a PC and can support multiple users.

Mainframe Computers:

Larger in size than mini computers. Mainframe computers have higher processing power can handle multiple processing tasks currently and supports a large number of users at a time.

Super Computers:

Super Computers are the fastest and are used in those areas which require extremely high speed processing and large storage capacities such as meteorology, energy, research, animated graphics as seen in the movie Jurassic Park. India has also manufactured Super Computers viz Pavan 9000 with speed 2.5×10^9 floating point operations (multiplication & division of real numbers). Per second PACE with speed 10×10^9 floating point operation per second etc.

25.4 CHARACTERISTICS OF A COMPUTER

Whatever may be the size of a computer it has the following basic characteristics.

(1) Speed: A computer works much faster than human beings. For example a present day PC can perform 1 million (10^6) multiplications in one second while a super computer can perform 20,000 millions (20×10^9) of multiplications in one second.

(2) Accuracy: A computer does exactly what it is instructed to do without any error as many time as required. If either the instructions or the data entered are incorrect then the results from the computer will be incorrect. However, while performing multiplication and division of big decimal numbers the results are accurate accurate upto 16, 32 and 64 significant figures depending on the processing power of the computer.

(3) Storage of Information:

A computer can store and retrieve a large amount of information depending on the storage capacity of the computer. For example a present day PC (with IG byte and disc) can store at about 500,000 pages of text. The information stored can be retained as long as desired by the user and can be retrieved as and when required and the retrieved information will be exactly in the form it was stored.

(4) Diligence:

A computer can work for several hours continuously without being tired and works with the same speed and accuracy throughout i.e. if a computer performs one million multiplications in one second with an accuracy of 16 significant figures then even after working continuously for 100 hours it can perform million multiplications in one second with 16 significant figure accuracy.

(5) Versatility:

A computer can immediately change from performing one type of task to another type of task at the command of the user. For example in one moment it may be processing examination scores and at the next moment it may play a game or may solve a set of equations etc.

(6) No Intelligence of its own:

In spite of all the above advantages the main limitation of a computer is that it does not have reasoning capabilities. It possesses no intelligence of its own. It works with the intelligence of the programmer and ...

25.5 THE COMPONENTS OF A COMPUTER SYSTEM

The ability of a computer to process data into information is due to the combined work of hardware and software. The Computer's machinery made up of electronic devices and circuits is the hardware where as human provided logic and instructions to the computer are software. Hardware can not do anything without software.

Hardware:

Computer hardware is composed of five units: input, central processing, internal memory, secondary storage and output. The input unit enables the computer to receive the data to be processed and the instructions necessary for processing. A commonly used input device is the Keyboard which types instructions and data into the computer. Other input devices are: mouse used for pointing at specific locations of the computer screen (Monitor), joystick used for games, scanner used for reading manuscripts.

The central processing units (CPU) is the place in a computer where the data is processed into information. It consists of two parts: Arithmetic - Logic unit (ALU) and Central Unit (CU). The actual processing is carried out in ALU as directed by the Control Unit. The CPU is located on a Computer Chip - a smaller piece of Silicon with millions of transitors hatched into it. The CPU Chip is plunged into the main computer circuit board called the Mother-board. The most

commonly used CPU chips in PC's - are 80386, 80486 and pentium manufactured by Intel Corporation of U.S.A.

A Computer's internal memory works in association with the CPU to store data and instructions necessary during processing. It is made up of chips plunged into the mother board. Two types of memory chips are used in a computer. They are Random Access Memory (RAM) and Read - only - Memory (ROM). Ram is available for storage only during the period the computer is powered on and the stored information in RAM is wiped out as soon as the computer is turned off. ROM is fixed in the computer during the time the compuer is manufactured and it is intended to store permanently the information required during the start up process. ROM also contains instructions to manage many computer operations such as providing the characters on the screen when a Key is pressed displaying the results of processing.

Because of volatility of Ram and limited internal memory (to lower the cost of computer as memory chips are expensive) some form of storage external to RAM is necessary to store data and programmes permanently. This form of storage is termed as secondary storage. Some of the secondary storage devices are hard disks, floppy disks, magnetic tape and CD - ROM (Compact disk - read only memory). When the internal memory sticks the information stored in secondary

storage under the direction of CU, the secondary storage locates the information, reads it, transfers it to internal memory. So secondary storage is slower than the internal memory but it has very high storage capacity as compared to internal memory.

The processed information is made available to the user through the output unit. The most common output devices are the video screen or monitor and the printer. The monitor saves immediately the data or instruction typed in the Key board. It also saves the processing in complete . When a permanent printed copy of the results is desired a printer is used to print the informations.

Secondary storage is a form of output when the processed information is stored in the secondary storage. It is also an input device when stored data are read from the disks or tapes. Another form of input/output is telecommunications - the transmission of data and information over some type of communication medium. Telecommunication links allow a computer to receive information from and transfer information to a computer situated at near by or far off places. Telecommunication links are established through the use of standard telephone lines and a modern (Modulate/ DE Modulate).

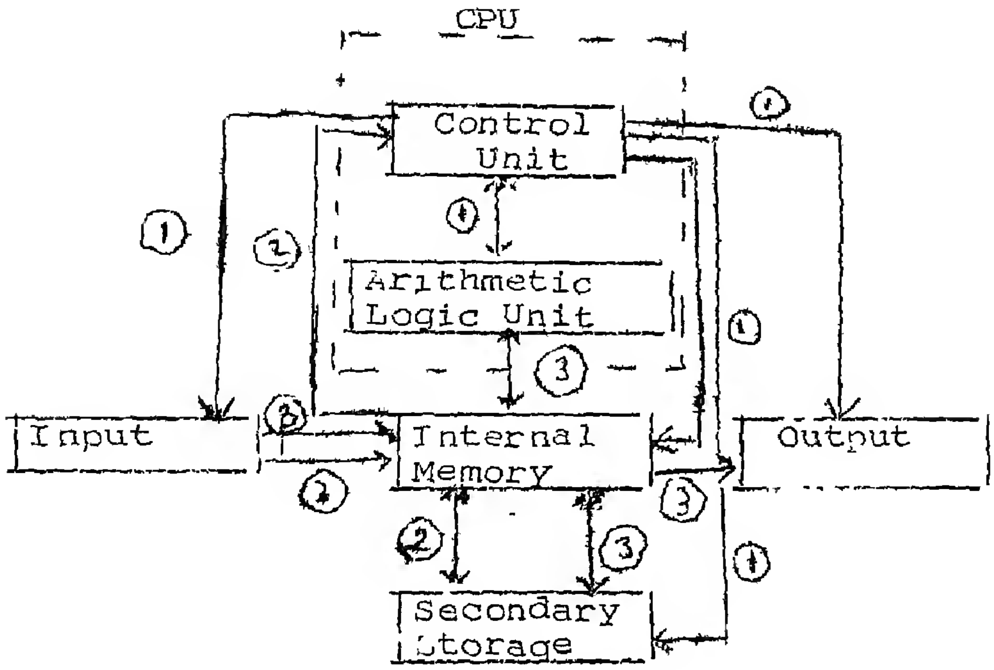
25.6 THE FUNCTIONING OF A COMPUTER

All processing taks are done by the CPU with the help of one or more of the three operation: (i) Calculating (adding, substracting etc.), Comparing (determining whether two items are same) and moving data items from one memory location to another.

The flow of data, instructions, control commands and results into within or out of the computer along with different units of a computer is shown schematically in fig. . The transfer of information within CPU and between CPU and internal memory is fast because they are either on chip or between chips. On the other hand transfer of data or information between input and output unit or secondary storage or internal memory is slow because these flows involve electro-mechanical devices like Key boards, disk drives and printers.

To execute a program, software stored in secondary storage is moved to the internal memory by giving appropriate instructions through the input device. Then from the internal memory the instructions are transferred to the control unit. Each program instruction is decoded by CU and then directs the activities of all other units. The control unit directs the input or secondary storage to send data to be processed to ALU through the internal memory and directs the ALU to process the data so received.

The ALU then executes the instructions and when they



1) Commands of CU

2) Flow of instructions

3) Flow of data or processed information

are completed the result is stored back in internal memory from where they are sent to output or secondary storage under the direction of CU.

Software:

The instructions given to the computer by the user or manufacturer is called the software. It directs the hardware to do the appropriate job. The Computer can not work without the software . There are two major types of softwares- the system software and the application software. The system software includes the operations system which co-ordinates the operations of the various

components to a computer system. The most popular operating systems are DOS (Disk Operating System) Windows and UNIX. The application softwares are prepared to be used in a computer for performing specific applications. They are commercially available in the form of software package. The software packages constitute programming disks and user mannuals which provide the description, how to use package. Several software packages are available in each of the application areas such as word processing, desk top publishing, games, graphics, spreadsheets, database management development tools etc. Some of the popular software packages are : Wordstar and Word perfect (Word process of package). Lotus 1 - 2 - 3 (Spread sheet) DBASE (data base management), Corel draw (graphics) and Ex (according to package).

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Number System

Number system play an important role in the design, organisation and understanding of computers. Number systems are basically of two types: Nonpositional and Positional .

26.1 Non-Positional Number Systems

In early days, human beings counted on fingers. When ten figure were not adequate stones, pebbles or sticks were used to indicate values. This method of counting uses an additive approach or the non-positional number system.

In this system, we have symbols such as I for 1, II for 2, III for 3, IIII for 4. Each symbol presents the same value regardless the same value regardless of its position in the number and the symbols are simply added to find out the value of a perticular number.

Since it is very difficult to perform arithmetic with such a number system, positional number systems were developed as the century passed.

26.2 Positional Number System

In a positional number system, there are only a few symbols called digits, and these symbols represent different values depending on the position they occupy in the number. The value of each digit in such a number is determined by three consideration:

1. the digit itself
2. the position of the digit in the number and
3. the base of the number system

26.3 Decimal Number System

In this system the ten symbols which are used are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and each of these is also known as a digit. In any number which we come across, these digits have a place value.

For example, the number 273, the place value of 2 is 200, the place value of 7 is 70 and the place value of 3 is 3. Hence we can write it as

$$273 = 2 \text{ hundreds} + 7 \text{ tens} + 3 \text{ ones.}$$
$$= 2 \times 100 + 7 \times 10 + 3 \times 1$$
$$= 2 \times 10^2 + 7 \times 10^1 + 3 \times 10^0$$

Here the 10 is called the base of the number system.

The general expression for representing a positive integer in the decimal system by using positional notation .3 is

$$a_{n-1}\ 10^{n-1} + a_{n-2}\ 10^{n-2} + \ldots\ldots + a_0$$

where a_{n-1}, a_{n-2} a_{n-3} ..... a_0 are digits (0 to 9) and n is the number of digits in the integer.

The base or radix, of a number system is defined as the number of different digits which can occur in each position in the number system. The decimal system has a base 10. Thus the system has 10 different digits, any one of which can be used in each position in a number.

A decimal number, for example 43.27 is represented in this notation as

$$(43.27)_{10} = 4 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-1} + 7 \times 10^{-2}$$

(10 is the base of the system)

26.4 Binary Number System

The system of numbers has become indispensable with the growing popularity of the computers. In this system we use only two digits, namely 0 and 1 and use the number 2 as the base of the system. A similar type of positional notation is used in the binary number system as in the decimal system. The decimal equivalent of a binary integer $(a_{n-1}\ a_{n-2} \ldots\ldots a_0)_2$ is $a_{n-1}\ 2^{n-1} + a_{n-2}\ 2^{n-2} + \ldots\ldots + a_0$

Where $a_{n-1}\ a_{n-2},\ a_{n-3} \ldots\ldots a_0$ are either 1 or 0 and n is the number of digits in the number.

In this system the base or radix is 2 and only 0 and 1 are the two different numerals that can be used in this system. For example

$$(101)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

Similarly a fractional number in Binary Number system can be represented as

$$(101.10)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0$$

'Binary digit' is often referred to by the common abbreviation 'bit'.

26.5 OCTAL NUMBER SYSTEM

The octal number system has a base of eight and hence 8 different symbols are used to represent numbers. These numbers are 0,1,2,3,4,5,6 & 7. The positional National system for this number system is written in the pavors of eight. The decimal equivalent of an octal number $(a_{n-1}\ a_{n-2}\ \ldots\ a_0)_8$ is

$$a_{n-1}\ 8^{n-1} + a_{n-2}\ 8^{n-2} + a_{n-3}\ 8^{n-3} + \ldots + a_0\ 8^0$$

where a_{n-1}, a_{n-2}, a_{n-3}, ... a_0 are digits 0 to 7 and n the number of digits of the octal number.

For example $(257)_8 = 2 \times 8^2 + 5 \times 8^1 + 7 \times 8^0$

similarly a fractional octal number is written as

$(25.16)_8 = 2 \times 8^1 + 5 \times 8^0 + 1 \times 8^{-1} + 6 \times 8^{-2}$

26.6 Hexadecimal Numbers:

This system of numbers is developed using the sixteen symbols 0,1,2,3,4,5,6,7,8,9, A, B,C,, D,E, F and the '16' is the base of this system. Note that A,B,C,D,E & F stand for 10, 11, 12, 13, 14 & 15 respectively.

The system of 16 numbers is known as the hexadecimal Number System. The positional notational form of this system is represented in the powers of 16. The general rule for its decimal equivalent is as follows:

$$a_{n-1}\ 16^{n-1} + a_{n-2}\ 16^{n-2} + \ldots + a_0.16^0$$

Where $a_{n-1}\ a_{n-2}\ \ldots\ldots\ a_0$ are the digits in the hexadecimal number.

For example

$$(1\ A\ F)_{16} = 1 \times 16^2 + A \times 16^1 + F \times 16^0$$

Similarly a fractional number is represented as follows:

$$(BB.8)_{16} = B \times 16^1 + B \times 16^0 + 8 \times 16^{-1}$$

26.7 Converting from one Number System to another

Numbers expressed in decimal are much more meaningful to us than are values expressed in any other number system. This is mainly because of the fact that we have been using decimal numbers in our day to day life right from childhood. However any number in one number system can be represented in any other numbers system. Because the input and the final output values are to be in decimal number system. Computer professionals are often required to convert numbers in other number systems to decimal and vice versa.

26.8 Converting to decimal from another Base

The following three steps are used to convert to a base 10 value from any other number system.

1st Step : Determine the positional value of each digit (depends on the position of the digit and the base of the number system).

2nd Step : Multiply the obtained positional value by the digits in the corresponding position.

3rd Step : Sum the products calculated in Step 2.

Example - 1 $(11011)_2 = (\ ?\ )_{10}$

Binary digits	1	1	0	1	1
Positional value	2^4	2^3	2^2	2^1	2^0

Required number in decimal system

$= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 16 + 8 + 0 + 2 + 1$

$= (27)_{10}$

$(11011)_2 = (27)_{10}$

Example - 2 $(\ 101\ .\ 11\ )_2 = (\ ?\ )_{10}$

Binary bits	1	0	1	1	1
Positional value	2^2	2^1	2^0	2^{-1}	2^{-2}

Required Number $= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$

$= 4 + 0 + 1 + 0.5 + 0.25$

$= (5.75)_{10}$

$(101.11)_2 = (5.75)_{10}$

Example - 3 $(4705)_8 = (\ ?\ )_{10}$

Octal digits	4	7	0	5
Positional value	8^3	8^2	8^1	8^0

Required number in Octal number system

$= 4 \times 8^3 + 7 \times 8^2 + 0 \times 8^1 + 5 \times 8^0$

$= 4 \times 512 + 7 \times 64 + 0 \times 8 + 5 \times 1$

$= 2048 + 448 + 0 + 5$

$= 2501$

$(4705)_8 = (2501)_{10}$

Example - 4 $(315.54)_8 = (\ ?\ )_{10}$

Octal digits	3	1	5	5	4
Positional value	8^2	8^1	8^0	8^{-1}	8^{-2}

Required number $= 3 \times 8^2 + 1 \times 8^1 + 5 \times 8^0 + 5 \times 8^{-1} + 4 \times 8^{-2}$

$= 3 \times 64 + 8 + 5 + \frac{5}{8} + \frac{4}{64}$

$= 192 + 8 + 5 + .625 + 0.0625$

$= (215.6875)_{10}$

$(315.54)_8 = (215.6875)_{10}$

Example - 5 $(IAC)_{16} = (\quad)_{10}$

Hexadecimal digits	I	A	C
Positional value	16^2	16^1	16^0

Required number in Hexadecimal number system

$= 1 \times 16^2 + 10 \times 16 + + 12 \times 1$

$= 256 + 160 + 12 = 428$

$(IAC)_{16} = (\ 428\ )_{10}$

Example - 6 $(2B.C4)_{16} = (\ ?\ )_{10}$

Hexadecimal digits	2	B	C	4
Positional value	16^1	16^0	16^{-1}	16^{-2}

Required number in Hexadecimal system

$= 2 \times 16^1 + 11 \times 16^0 + 12 \times 16^{-1} - 14 \times 16^{-2}$

$= 2 \times 16 + 11 \times 1 + \frac{12}{16} + \frac{4}{256}$

$= 32 + 11 + 0.75 + 0.015625$

$= (43.765652)_{10}$

$(2B.C4)_{16} = (43.765652)_{10}$

26.9 **Converting from base 10 to a new base**

(Division - Remainder Technique)

The following four steps are used to convert a number from base 10 to a new base;

Step - 1 Divide the decimal number to be converted by the value of the new base.

Step - 2 Record the remainder from step 1 as the right most digit (least significant digit) of the new base number.

Step - 3 Divide the quotient of the previous division by the new base.

Step - 4 Record the remainder from step 3 as the next digit (to the left) of the new base number.

Repeat steps 3 and 4 recording remainders from right to left, until the quotient becomes zero in step 3. Note that the last remainder thus obtained will be the most significant digit (M.S.D.) of the new base number.

Example - 1 $(25)_{10}$ $(\quad)_2$

Step 1 and 2 : 25 ÷ 2 = 12 and Remainder 1

Step 3 and 4 : 12 ÷ 2 = 6 and remainder 0

Step 3 and 4 : 6 ÷ 2 = 3 and remainder 0

Step 3 and 4 : 3 ÷ 2 = 1 and remainder 1

Step 3 and 4 : 1 ÷ 2 = 0 and remainder 1

Here first remainder 1 becomes the last significant digit (L S D)and the last remainder becomes the most significant digit (MSD)

Alternatively

		Remainder (Binary)
2	25	
2	12	 1
2	6	 0
2	3	 0
2	1	 1
	0	 1

Hence $4(25)_{10} = (10011)_2$

<u>Example - 2</u> $(952)_{10} = (\ ?\ )_8$

		Remainder (Octal)
8	952	
8	119	 0
8	14	 7
8	1	 6
	0	 1

Hence $(952)_{10} = (1670)_8$

<u>Example - 3</u> $(428)_{10} = (\ ?\ )_{16}$

		Remainder(Hexa decimal)
16	428	
16	26	12 = C
16	1	10 = A
	0	1 = 1

Hence $(428)_{10} = (IAC)_{16}$

COMPUTER ARITHMETIC

Number systems play an important role in the design, organisation and understanding of computers. An important feature of the computer is that the data processing in a computer is based on a number system different from our decimal system. The operations involved in Arithmetic can also be logically calculated in A.L.U. of Computers. Arithmetic in Binary system is now be discussed as given below.

27.1 Arithmetic in Binary System

Binary Addition:

The following are the rules of binary Addition.

x	y	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

	x	Augend
	+ y	Addend
Sum		With or without carry

Here 'x' and 'y' are the two digits that are to be added. A carry might occur or not as shown above.

Example:-1 $(1011)_2 + (1000)_2 = (\ ?\ )_2$

$$\begin{array}{rl} 1\ 0\ 1\ 1 & \text{Augend} \\ 1\ 0\ 0\ 0 & \text{Addend} \\ \hline \text{Carry } \boxed{1}\ \ 0\ 0\ 1\ 1 & \end{array}$$

As in the decimal system, the addition in the binary system also starts from the least significant bits. If a carry occurs it is taken to the next significant bit as so on.

Example - 2 $(11111)_2 + (11010)_2$ (?)

	1 1	Carry
	1 1 1 1 1	Augend
	1 1 0 1 0	Addend
1	1 1 0 0 1	Sum

Hence $(11111)_2 + (11010)_2 = (111001)_2$

Example - 3 $(1101.101)_2 + (1001.010)_2$

1 1	Carry
1 1 0 1 . 101	Augend
1 0 0 1 . 010	Addend
1 0 1 1 0 . 111	

Hence $(1101.101)_2 + (1001.010)_2$

$= (10110.111)_2$

Subtraction:

Subtraction in the binary system is done as in the decimal system i.e. by borrowing concept. In decimal system if digit being subtracted is greater than the digit from which it is subtracted, then one value is borrowed from next digit on the L.H.S. and taken as 10. In case of binary system the borrowed digit is 2 only.

The rules of subtraction are as under.

x	y	Diff.	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

x - Minuend

y - Subtrahend

Difference

Let us consider some examples to subtract a binary number from another.

Example - 1 $(11\ 10\ 11)_2 - (\ 11110\ )_2$

Borrow	1 1 0 0
Minuend	1 1 1 0 1 1
Subtrahend	0 1 1 1 1 0
Difference	1 1 1 0 1

$$(111011)_2 - (11110)_2 = (11101)_2$$

Example - 2 $(\ 1.001\ )_2 - (\ 0.110\ )_2 = (\quad)_2$

1 1	-	Borrow
1. 0 0 1	-	Minuend
0. 1 1 0	-	Subtrahend
0. 0 1 1	-	Difference

In all above examples we have seen that the subtrahend is smaller than the minuend. So the subtraction using above procedure was easy. But if the subtrahend is larger than minuend the procedure lead to incorrect values. If this was the case in decimal system you would reverse the minuend and subtrahend and put a negative sign to the magnitude . So obtained. But it is not so in binary system. Here even the sign bit is represented by '0' and '1'.

To manipulate negative numbers the complementary method is isoled. Both 1's and 2's complement method can be used. In order to understand complementary subtraction, it is necessary to know what is meant by the complement of a number. For a number which has 'n' digits

in it, a complement is difined as the difference between the number and the base raised to the n th power minus one.

Example-1 Find the complement of 37_{10}. Since the number has 2 digits and the value of base is 10.

So $(\text{Base})^n - 1 = 10^2 - 1 = 99$

Now $99 - 37 = 62$

Thus the complement of $37_{10} = 62_{10}$

Example-2 Find the complement of $(10101)_2$

Since the number has 5 digits and the value of base is 2.

So $(\text{Base})^n - 1 = 2^5 - 1 = 31_{10}$

Also $(10101)_2 = (21)_{10}$

Now $31_{10} - 21_{10} = 10_{10} = 1010_2$

Thus the complement of $(10101)_2 = (01010)_2$

We can observe from the above example that in case of binary numbers, it is necessary to go through to the usual process of obtaining complement. Instead, when dealing with binary numbers a quick way to obtain a number's complement is to transform all its 0's to 1's and all its 1's to 0's. For example, the complement of 1011010 is 0100101. Subtraction by the complement method may involve the following three steps.

Step-1 Find the complement of the number you are subtracting (Subtrahend).

Step-2 Add this to the number from which you are taking away (minuend).

Step-3 If there is a carry of 1, add it to obtain the result. If there is no carry recomplement the sum and attach a negative sign to obtain the result.

Example-1 Subtract 56_{10} from 92_{10} using complementary method.

Step-1 Complement of 56_{10}

$= 10^2 - 1 - 56$

$= 99 - 56 = (43)_{10}$

Step-2

$$\begin{array}{r} 92 \\ +\ 43 \\ \hline \boxed{1}\ 35 \end{array}$$

Carry 1 — Complement of 56

Step-3 ...... Add the carry of 1

$= 36$

Example-2 Subtract $(35)_{10}$ from $(18)_{10}$ using complementary method.

Step-1 Complement of $35_{10} = 10^2 - 1 - 35$

$= (64)_{10}$

Step-2

$$\begin{array}{r} 18 \\ +\ 64 \\ \hline 82 \end{array}$$

(Complement of $(35)_{10}$

Step-3 There is no carry. So recomplement the sum and attach a negative sign to obtain the result.

Result $= -(99 - 82) = -17$

Let us rework these examples using binary numbers.

Example-3 Subtract 0111000_2 $(56)_{10}$ from $(1011100)_2$ $(92)_{10}$ using complementary method.

$$
\begin{array}{rl}
1011100 & \\
1000111 & \text{(Complement of 0111000)} \\
\hline
\boxed{1}\ 0100011 & \\
1 & \text{(Add the carry of 1)} \\
\hline
100100 &
\end{array}
$$

Result $0100100_2 = 36_{10}$

Example-4 Subtract $(100011)_2$ 35_{10} from $(010010)_2$ 18_{10} using complementary method.

$$
\begin{array}{rl}
010010 & \\
+\ 011100 & \text{(Complement of 100011)} \\
\hline
101110 &
\end{array}
$$

As there is no carry, so we have to complement the sum and attach a negative sign to it. Hence

Result = $-\ 010001_2$

(Complement of 101110_2) $= -\ 17_{10}$

Binary multiplication

Binary multiplication is very simple and is similar to the decimal multiplication. The rules for multiplying the 4 combinations of binary digits is given below.

x	y	x X y
0	0	0
0	1	0
1	0	0
1	1	1

x - multiplicand

y - multiplier

x X y = Produce

Let us consider some examples to understand the application of the above rules.

<u>Example-1</u> $(11001)_2 \times (101)_2 = (\quad)_2$

```
    11001       Multiplicand
      101       Multiplier
  -------
    11001
   00000        Partial products
  11001
  -------
  1111101       Products
```

$(11001)_2 \times (101)_2 = (1111101)_2$

<u>Example-2</u> $(11.1)_2 \times (1o1)_2 = (\ ?\ )_2$

```
     11 . 1     Multiplicand
      1 . 1     Multiplier
    -------
      1 1 1
    1 1 1       Partial products
    -------
  1 0 1 0 1     Product
```

The decimal point is placed before 2 digits as in decimal system.

$(11.1)_2 \times (1.1)_2 = (101.01)_2$

27.2 <u>Binary Division</u>

The method used for division in decimal system is used even in Binary division. These are no hard and fast rules in the form of table as in the previous cases. Let us consider an example to understand in more detail.

<u>Example-1</u> $(10110)_2 \div (11)_2 = (\quad)_2$

```
    10110       Dividend
       11       Divisor
```

```
11 ) 1 0 1 1 0 ( 111        Quotient
    -1 1
    ------
      1 0 1
    -   1 1
    ------
        1 0 0
    -     1 1
    ------
            1        Remainder
```

Example-2 Calculate 11001 ÷ 101

```
        Binary                      Decimal

101 )1 1 0 0 1 ( 101            5 ) 25 ( 5
    -1 0 1                           25
    ---------                       ----
         1 0 1                        0
       - 1 0 1
    ---------
           0
```

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28.1 Approach to Problem Solving

Algorithms and Flow Charts

A computer has no intelligence of its own. Its I.Q is Zero. It performs a certain function faithfully at a very fast speed, but can not take any decisions of its own. The solution to various problems are brought out by computers with the help of various programs which control 'intelligence' conferred on them by the programmer. A list of instructions telling the computer 'What to do' is called a program. The program is fed in to the computer through mechanical electromagnetic or photoelectric means. Some programs control the

basic operations of the computer and are common for all type of uses of a perticular computer. They therefore are permanently stored in its memory (ROM). Other set of instructions which guides the computer for execution a specific job have to be specially written in form of program in a specific manner. Each program is to be prepared very carefully in a sequential and clear manner. The problem is to be studied carefully and analysed, and then a sequence of elementary instructions is to be drawn up in such a manner that it is obeyed literally and the problem will be faithfully solved. Such a set of instruction is called an algorithm.

Algorithms:- An algorithm may be formally defined as an ord red sequence of well defined and effective operations which perform a task or computation and terminate in a finite amount of time. Thus in order to qualify as an algorithm, a sequence of instructions must posses the following characteristics:

(i) The instructions must be ordered. This means that after execution of any step the next step of execution is clearly evident.

(ii) Each and every instruction should be precise and un-ambiguous to the person executing the instruction.

(iii) The instructions must be effective. This means that some formal method must exist for carrying out the operation and getting the result.

For example find $\sqrt{3}$ accurate to three decimal places is an effective operation where as find exact value of $\sqrt{3}$ or find out if there is God are not effective. To gain insight in to algorithms. Let us consider simple examples.

<u>Example-1</u> Write an algorithm to find the area of a triangle whose sides are given.

We know that area of A,

$$A = \sqrt{S(S-a)\ (S-b)\ (S-c)}$$

Where $S = \frac{a + b + c}{2}$ and a, b & c are the measure of its three sides.

Algorithm: A little study of the problem will reveal following steps.

(1) Get the length of the sides a, b & c

(2) Calculate $S = \frac{a + b + c}{2}$

(3) Calculate $A = \sqrt{S(S-a)\ (S-b)\ (S-c)}$

(4) Write the value of A

(5) Stop.

<u>Example-2</u> There are 50 students in a class who appeared in their final examination. Their mark sheets have been given to you. Write an algorithm to calculate and print the total number of students who passed in first division.

<u>Algorithm</u>

Step-1 Initialize total first division as 1 or zero depending on whether the 1st student has obtained 1st division or not and total mark sheets checked as 1.

Step-2 Take the mark sheet of the next student.

Step-3 Check the division column of the mark sheet to see if it is 1: if no, go to Step 5.

Step-4 Add 1 to total first division.

Step-5 Add 1 to total mark sheets checked.

Step-6 Is total mark sheet checked = 50 if no go to step 2.

Step-7 Print total first division.

Step-8 Stop

In the above examples the algorithms are described using ordinary English. But often it is described by the use of diagrams called flow charts. Thus a flow chart is a diagramatic representation of an algorithm.

28.2 Why Flow Charts ?

- Flow chart are drawn up as a pictorial guide by programmers for planning the procedure for the solution of a problem.
- It helps a person to understand at a glance the sequence of steps necessary to solve a given problem.
- A flow chart indicates the direction of flow of a process, relevant operations and computations, points of decisions and other information which is a part of solution.
- Properly developed and checked, the flow chart provides excellent guide for writing the program.

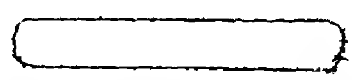
- Flow charts also facilitate commuuication among the programmer and the authorities.
- They also help in debugging the program i.e. in finding out the mistakes in the program.

Flow Chart Symbols:

For easy visual recognition a standard convention is used in drawing flow charts. Some symbols are needed to indicate the necessary operations in a flow charts which have been standardised by the American National Standards Institute (ANSI).

Terminal

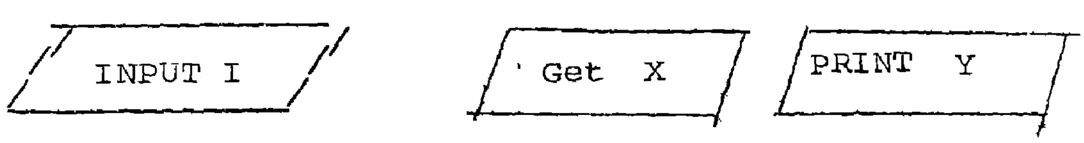
The terminal symbol, as the name implies, is used to indicate the beginning (START), ending (STOP) and Pauses (HALT) in the logic flow. It is the first and the last symbol in the program logic. In addition, if the program logic calls for a pause in the program, that also is indicated with a terminal symbol.

Terminal Symbol ⟶ 

(Rectangle with rounded sides)

Input and Output Indicators

Parallelograms are used to represent input and output operations. They are used as in the following examples:



Process - indicators

A processing symbol is used in a flow chart to represent arithmetic and data movement instructions. A rectangle is used to indicate any operation or arithmetic

an assignment operation as shown in the following examples:

X = 2	A = Y + Z
(Assignment)	(Arithmatic operation)

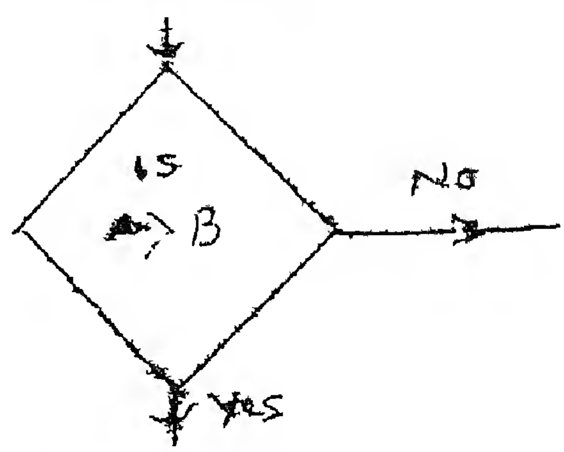
Flow lines

An arrow (↓↑ ——→ ←——) is used to indicate the direction of flow of instruction. Every line in a flow chart must have arrow on it.

ASSIGNMENT: The symbol A ← B refers to assigning B to A i.e. A is replaced by B.

28.3 Decision Makers:

The diamond ◇ is used for indicating the step of decision making and therefore known as decision - box and is the condition to be tested inside it. The further flow path is selected by Computer based on the test condition being true or false. Thus decision box which is normally entered from top exited by different process paths. The decision box must have two exists.



Two way Branch

28.4 **Connector:**

A circle ◯ is used to join different parts of a flow chart as a connector. The use of connectors gives a neat appearance to a flow chart. If the flow charts extends over more than one page, the different parts are joined with a connector. The fact that two joints are to be joined is indicated by enclosing them in circles and by writing the same identifying letter or digit inside both the circles. The connectors symbol may also be used to eliminate the crossing lines between one part of the flow chart to another.

The flow chart for any algorithm can be built from a combination of just the following three basic kinds of flow charts:

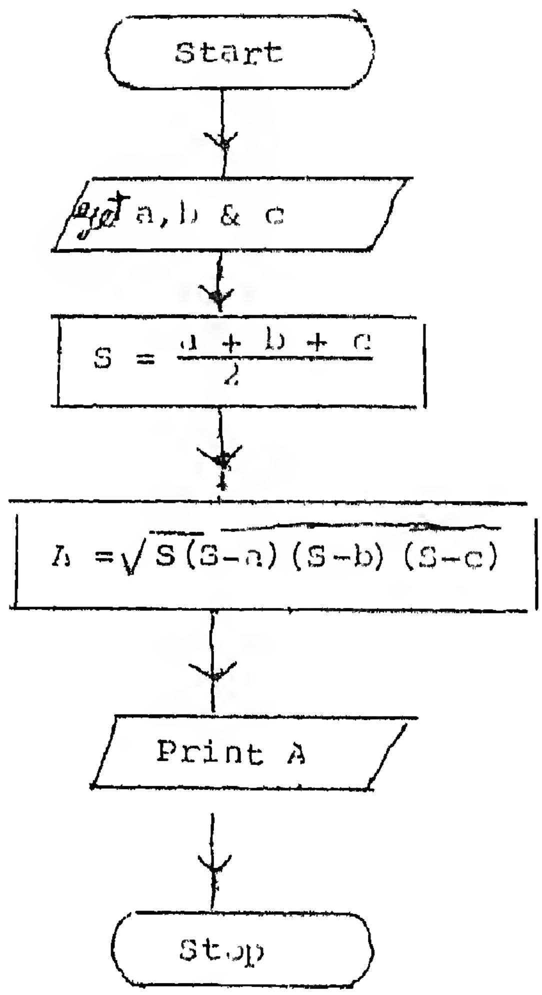
(i) Simple Sequence:

This consists of a set of independent instructions which are to be followed one after another from beginning to end. Let us take an example of making of tea.

i.e. First we have to put the stove on, take utensil and put it on the stove. Then fill it with a cup of water which is further mixed with milk, sugar and tea. Then it is boiled filtered and poured in various cups. The stove is then turned off.

As another example we draw the flow chart of algorithm 1 as follows:

Flow Chart

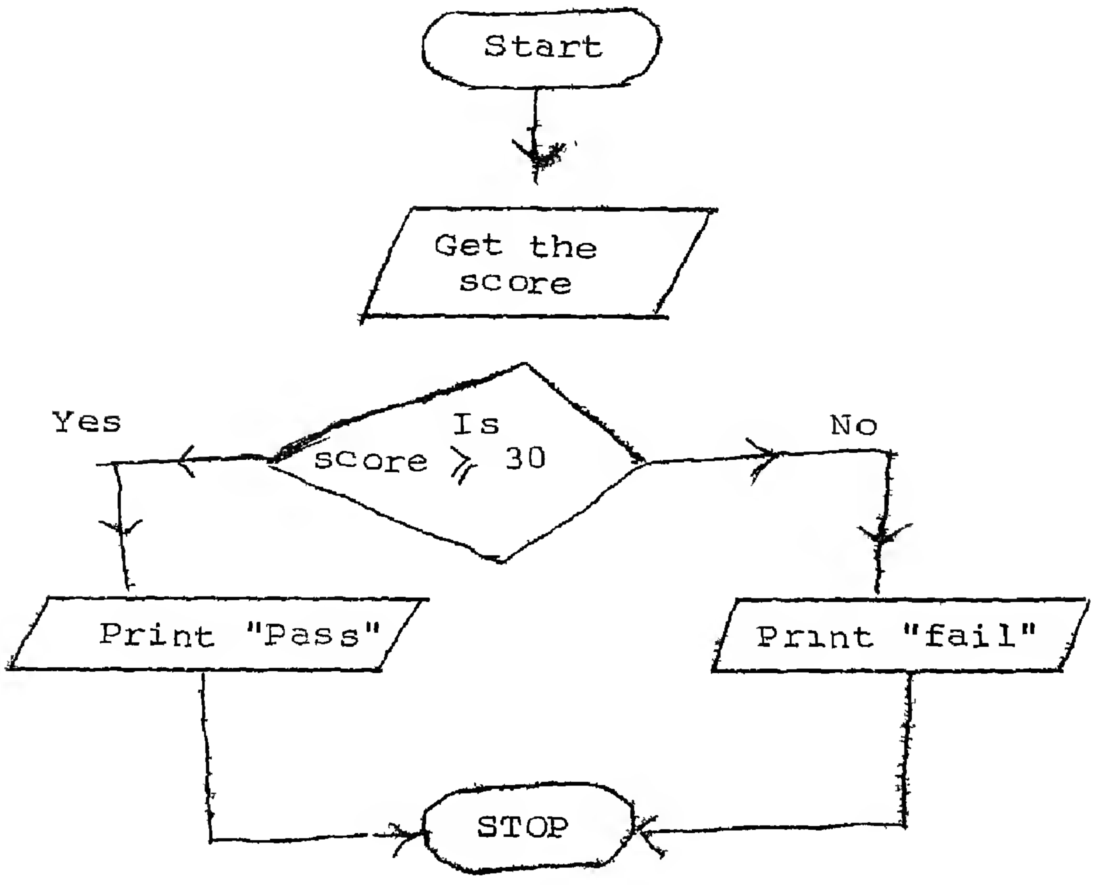


Simple Sequeutial Flow Chart

28.5 Simple Logical Selection (Decision Making)

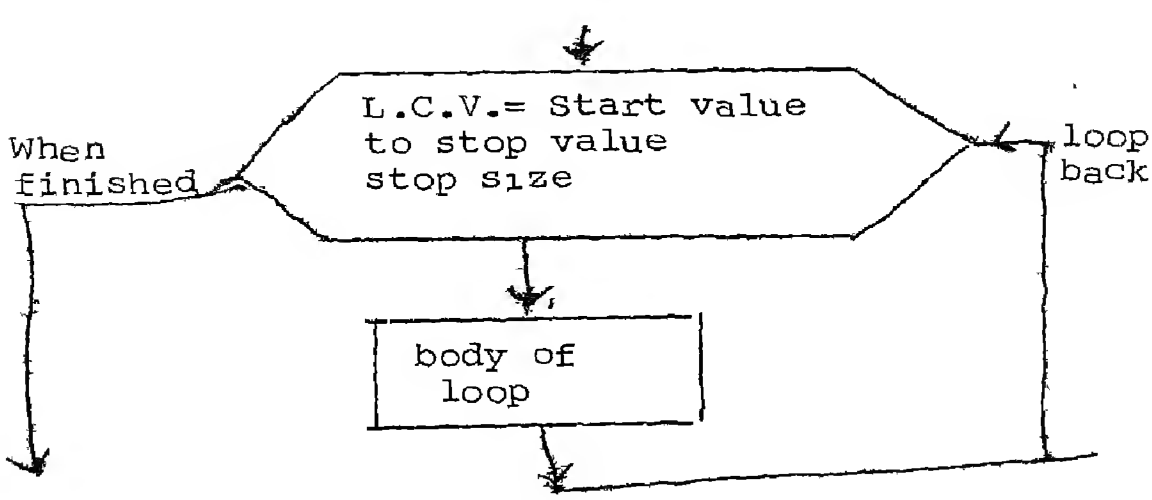
In sequentially structured instructions the instructions are executed in the order in which they appear from the begining till the end. A second important structure is the one that allows branching i.e. taking different paths depending on the results of a test condition given by one of the instructions as illustrated in the following example.

Example: Construct a flow chart to read a test score and classify as pass or fail, if pass mark is $\geqslant$ 30.



28.6 Simple repetition (Looping)

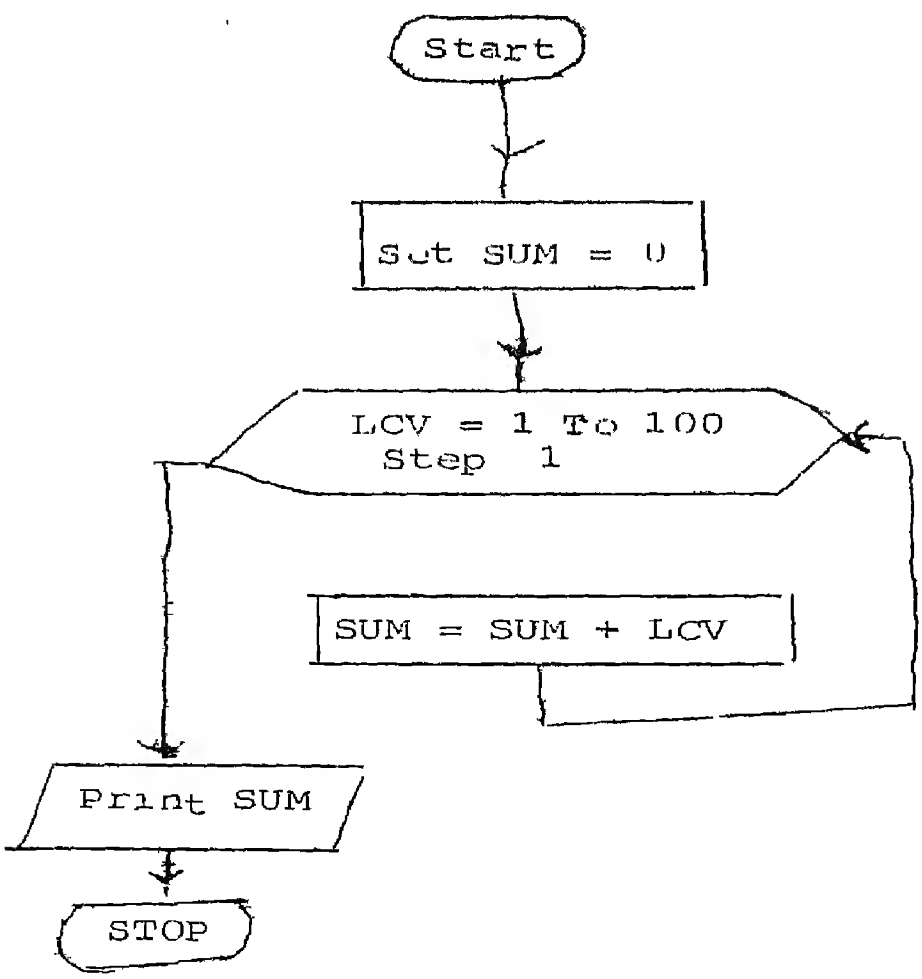
Sometimes we need to perform same operation several times. A structure that allows cycling through a block of instructions many times is called a loop. In one type of loop, called counted loops, the loops execute for a pre-determined, number of interactions. The flow chart symbol for this type of loop is



Inside the hexagon the start and stop values of the loop control variable (LCV) are given and also the step size is mentioned which is used to increment the LCV each time the body of the loop (a set of instructions to be repeatedly executed) is executed once. When the loop finishes the first instruction outside the loop is executed.

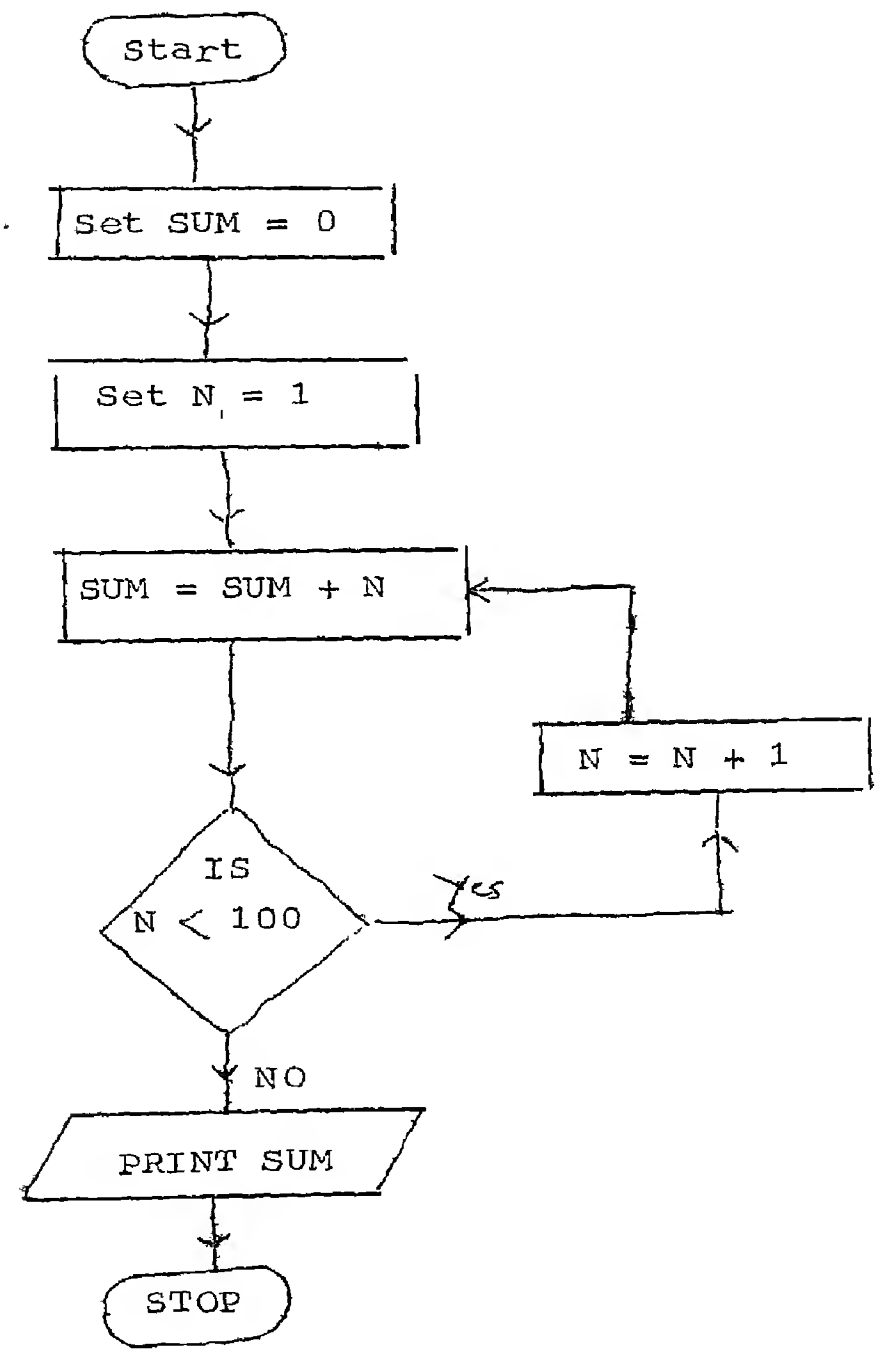
<u>Example</u> Draw a flow chart to find the sum of first hundred natural numbers.

Here the loop control variable (LCV) starts with 1, incremented by 1 and the repetition of execution of body of the loop continues until LCV $\nless$ 100.



Remark:

Looping can also be done by using logical selection. The following flow chart of the above example illustrates the method.



28.7 Flow Charting Rules

While programmers have a good deal of freedom in creating flow charts, there are a number ofgeneral rules and guide lines recommended by the American National Standards Institute (ANSI) to help standardize the

flow charting process. Various Computer manufacturers and data processing departments usually have similar flow charting standards. Some of these rules and guidelines are as follows.

(1) First chart the main line of logic, then incorporate detail.

(2) Maintain a consistent level of detail for a given flow chart.

(3) Do not chart every detail or the flow chart will only be a graphic representation, step by step of the program. A reader who is interested in greater details can refer to the program itself.

(4) Words in the flowchart symbols should be common statements and easy to understand. It is recommended to use descriptive titles written in designer's own language rather than in machine oriented language.

(5) Be consistent in using names and variables in the flow chart.

(6) Go from left to write and top to bottom in constructing flow charts.

(7) Keep the flow chart as simple as possible. The crossing of flow lines should be avoided as far as practicable.

(8) If a new flow charting page is needed, it is recommended that the flow chart be broken at an input or output point. Moreover properly labeled connectors should be used to link the portions of the flow chart on different pages.

28.8 Some Common Algorithms

1. Write an algorithm to find the solution of the equation $ax^2 + bx + c = 0$, where $a \neq 0$.

Solutions: 1. Obtain the values of a, b & c

2. Calculate $D = b^2 - 4ac$

3. If $D \geq 0$

then (i) Calculate $\sqrt{D}$

(ii) Calculate $\alpha = \frac{-b + \sqrt{D}}{2a}$

(iii) Calculate $\beta = \frac{-b - \sqrt{D}}{2a}$

(iv) Write α & β as roots

(v) Stop

else (i) Write: The equation has no real root.

(ii) Stop

2. Write an algorithm for finding the consistency of the following two simultaneous equations.

$$a_1 x + b_1 y + c_1 = 0$$

$$a_2 x + b_2 y + c_2 = 0$$

Solutions: 1. Get the values of a_1, b_1, c_1 and a_2, b_2, c_2.

2. Find $\frac{a_1}{a_2}$

3. Find $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$

4. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then write "The given system is consistent".

Stop

5. (i) If $\frac{a_1}{a_2} = \frac{c_1}{c_2}$

Then write "The given system is consistent.

Stop

(ii) else write the given system is inconsistent

Stop

3. Write an algorithm to find the percentage loss or gain when the cost price and selling price of an item is given.

<u>Solution</u>: 1. Obtain the values of C.P & S.P

2. If S.P. = C.P. then write "No Loss No gain"

3.(a) If S.P. > C.P. then

(i) Find profit = S.P - C.P

(ii) Find percentage profit =

$$P = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

(iii) Write "Percentage Profit"

Stop

(a) else

(i) Find Loss = C.P - S.P

(ii) Find % Loss = $\frac{\text{Loss}}{\text{C.P.}} \times 100$

(iii) Print % Loss

Stop

4. Write an algorithm to obtain the value of $|x|$ when x is known.

<u>Solution</u>: 1. Obtain the value of x

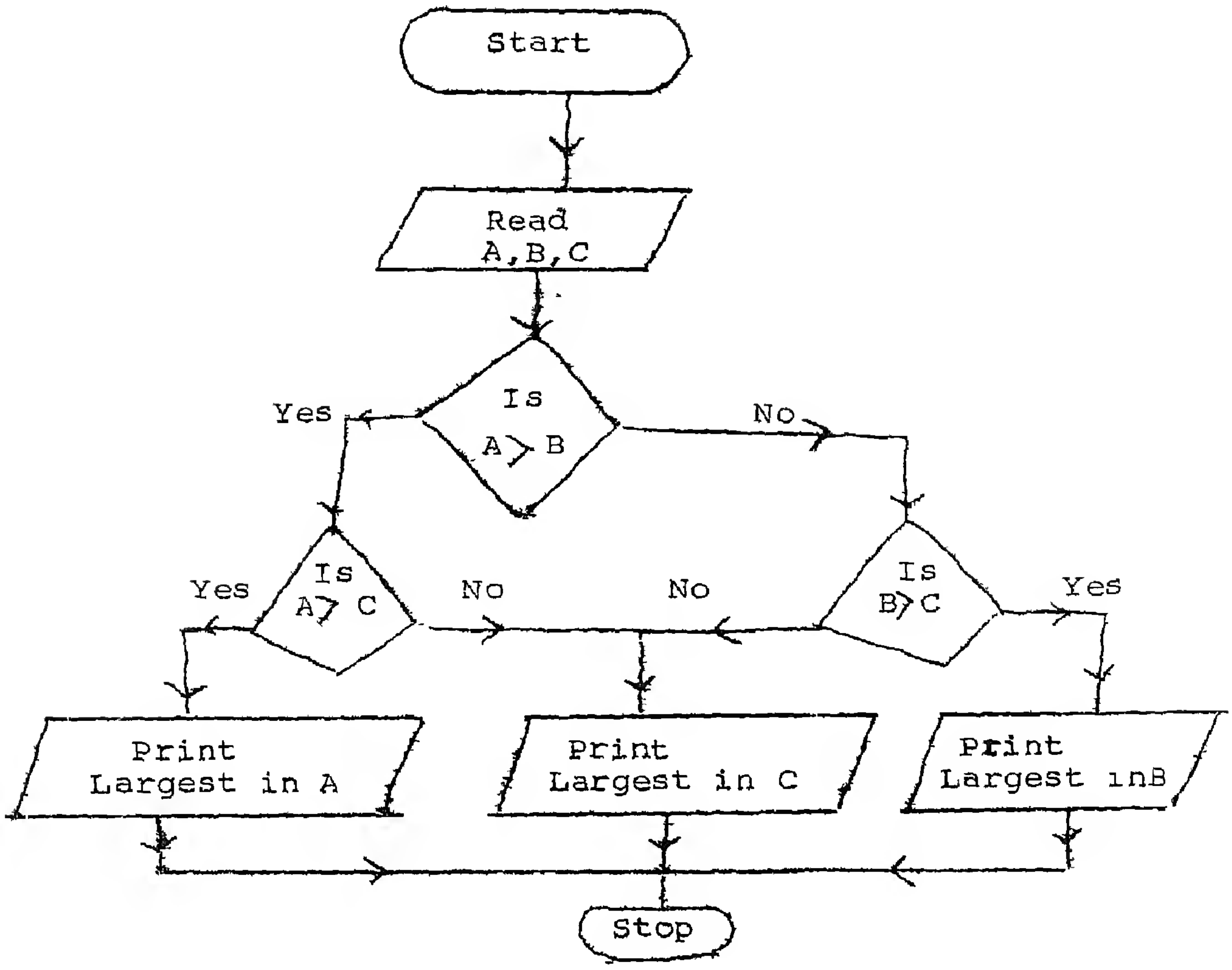
2. If x $\geq$ 0 then write $|x| = x$

Stop

else write $|x| = -x$

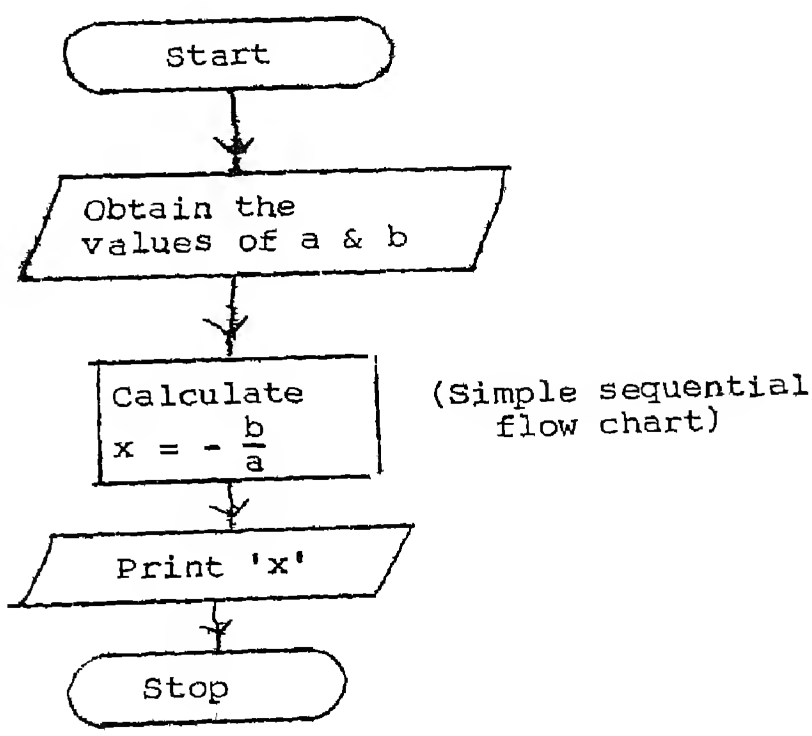
Stop .

2. Draw a flow chart to pick up largest of any three given numbers.

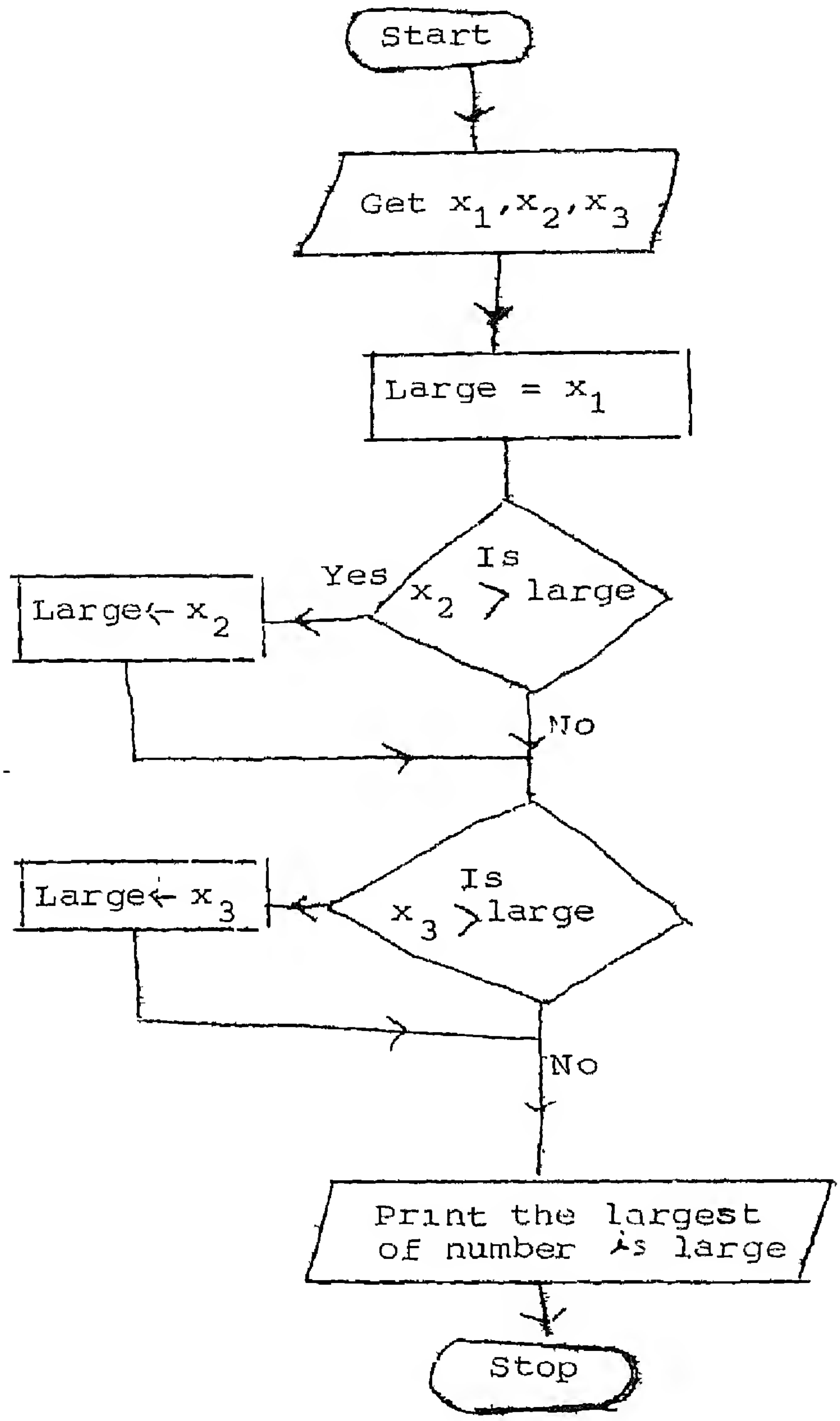


28.9 Some Common Flow Charts

1. Draw a flow chart to find the solution of the equation $ax + b = 0$, where $a \neq 0$



Alternative flow chart for 2.



Remark: The above algorithm can be generalised to find the largest of n given numbers.

Algorithm

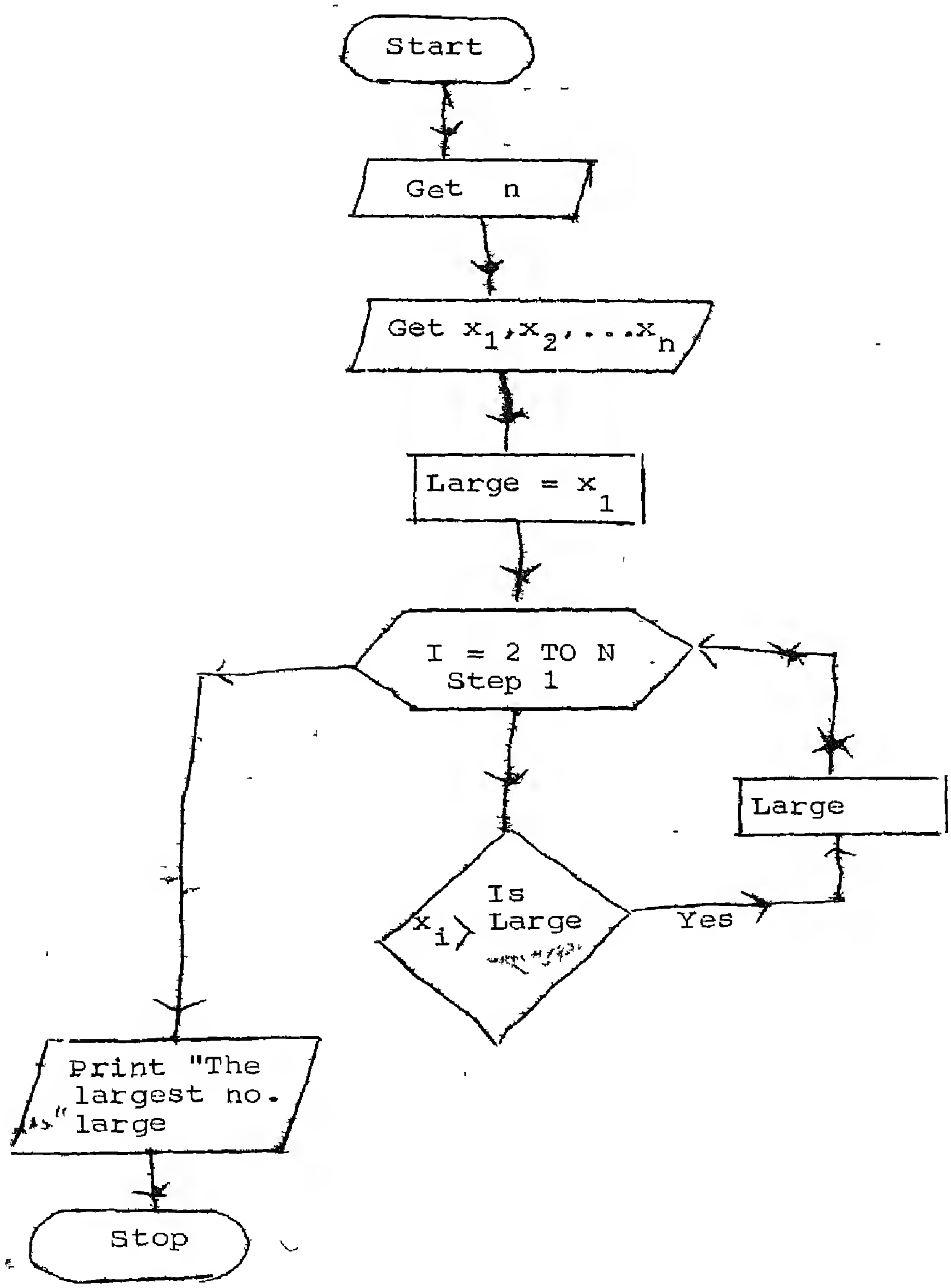
1. Get n
2. Get the numbers $x_1, x_2, \ldots, x_n$
3. Set Large = x_1

4. For i = 2, 3,......, n repeat the operation:
   If x_i > large then replace large by x_i
   else keep large as such .

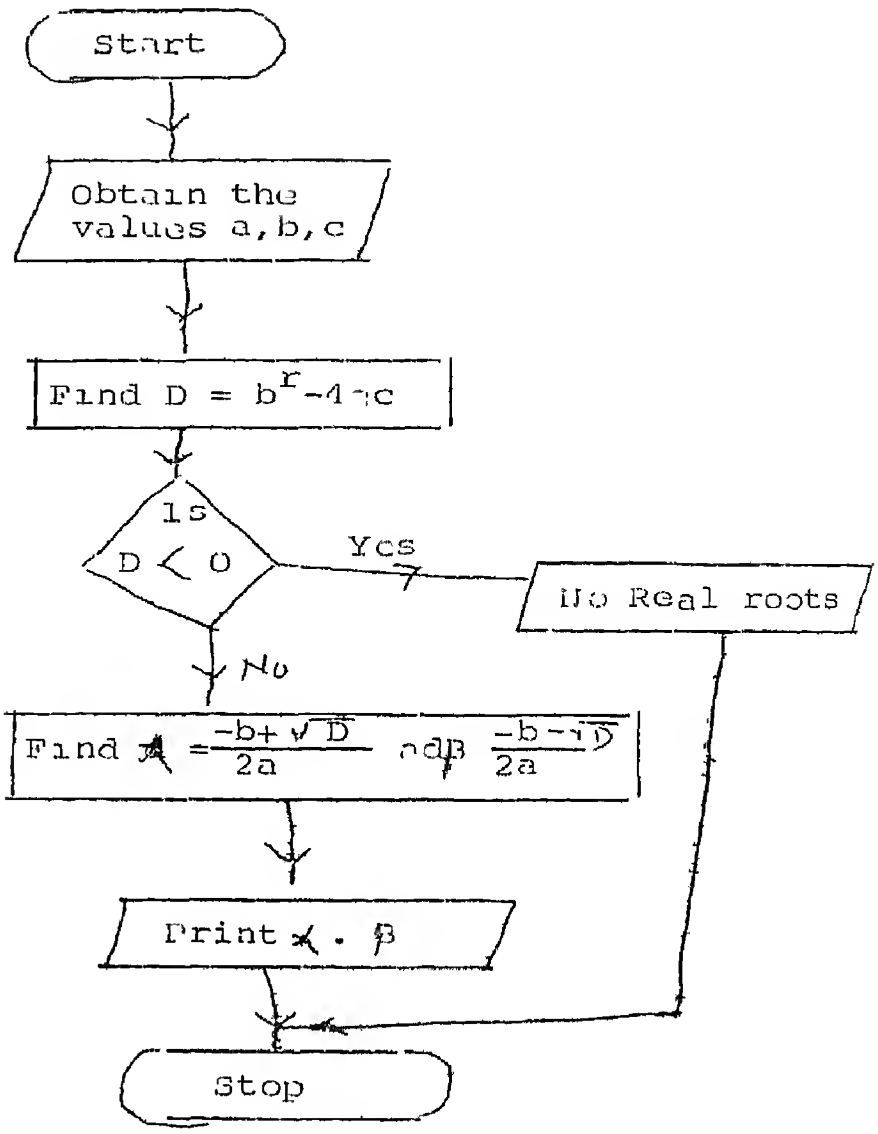
5. Write "The largest number is", large

   END

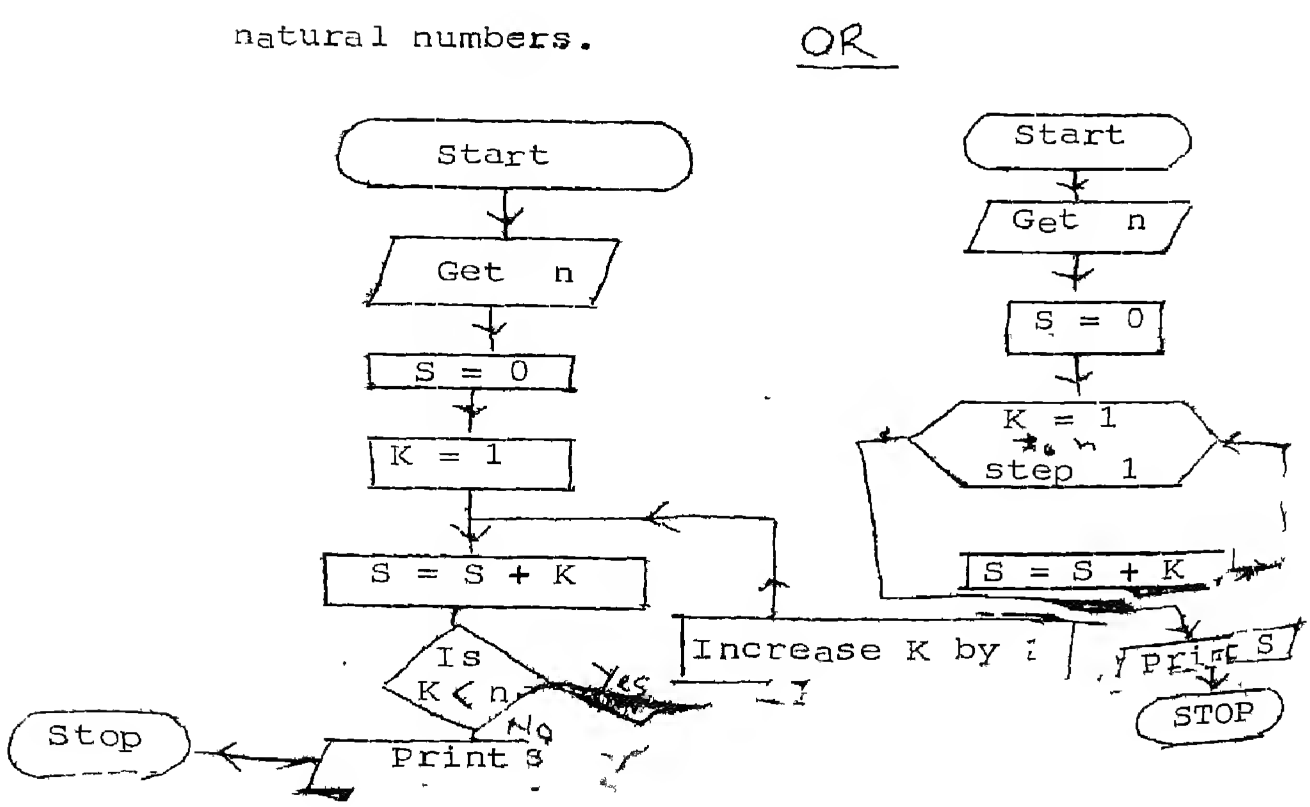
   The flow chart of the above algorithm is the following.



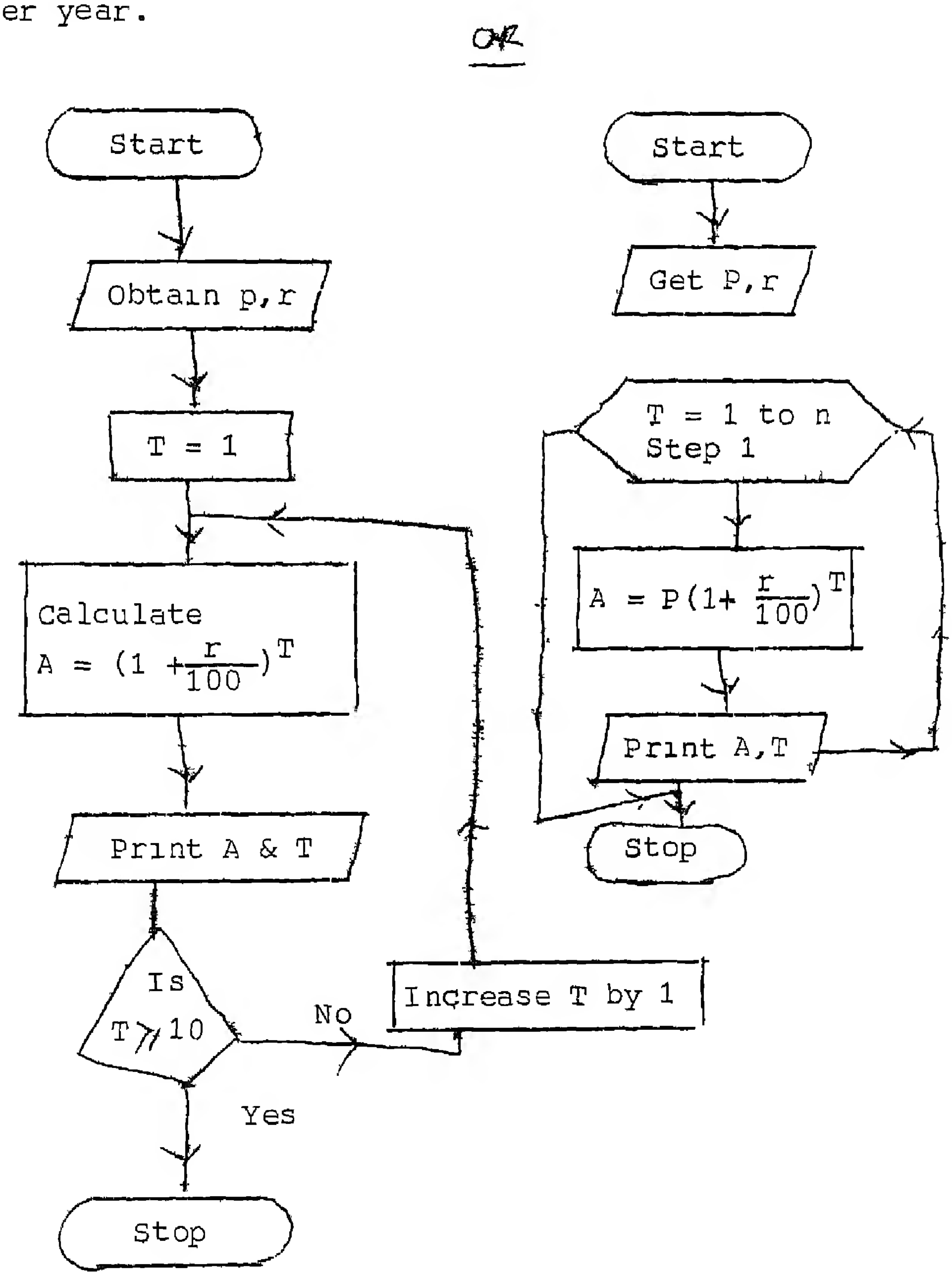
3. Draw a flow chart in order to find the real roots of the equation $ax^2 + bx + c = 0$ $(a, b, c \in R, a \neq 0)$



4. Draw a flow chart to find the sum of first n natural numbers.



5. Draw a flow chart to obtain the amount for 10 consecutive years starting from the time one year after, interest becomes applicable, given the principal P and rate of interest as r% per year.



APPENDIX - I

List of schools which participated in the identification of HARD SPOTS.

1. Govt. High School, Unit-5, Bhubaneswar.
2. Mahendra High School, Athamallik
3. Govt. High School, Jaleswar
4. Govt. High School (Boys), Titilagarh.
5. Govt. High School, Unit-VIII, Bhubaneswar.
6. Govt. High School, Mohangiri
7. B.B. High School, Dhenkanal
8. Mahatab High School, Delang.
9. Govt. High School (Girls), Kesinga.
10. Govt. High School, Unit-I, Bhubaneswar.
11. Bhuban High School, Bhuban.
12. M.M. High School, Pattamundai.
13. G.N. Vidyapitha, Sidhaswarpur
14. Govt. High School (Girls) Athagarh.
15. Jagannath Govt. High School Kamarda.
16. Capital High School (Girls) Unit-II, Bhubaneswar.
17. K.C. High School, Neelgiri
18. Govt. High School, Tangi.
19. Govt. High School (Girls) Badambadi, Cuttack
20. Kendrapara High School (Govt.), Kendrapara.
21. Bhagwati Vidyapitha, Sukleswar.
22. Govt. High School (Boys), Tarbha.
23. Govt. High School (Girls) Unit-VI, Bhubaneswar.
24. Bhargavi High School, Birnarsinghpur.
25. National Govt. High School Nuapada
26. Jubaraj High School, Talcher.
27. S.M. High School, Soro
28. Govt. High School (Girls), Tigiria.
29. Govt. High School (Girls), Banpur.
30. Govt. High School, Mendhasal.
31. Govt. High School, Unit-II, Bhubaneswar.
32. S.R. High School, Baliapal
33. P.M. Vidyapith, Tiglila
34. B.M. High School, Bhawanipatna
35. Govt. High School, J.N. Bidyapitha, Chaudakulat.
36. Govt. High School (Girls), Pallisahi.
37. B.N. Vidyapitha, Athagarh.
38. Girls High School, Narsinghpur.

39. Daspalla High School,
Daspalla.

40. Govt. High School,
Sahidnagar, Bhubaneswar

41. S.B. High School,
Bhingharpur

42. R.C. High School,
Khandapur.

43. Govt. High School(Boys)
Unit-VIII, Bhubaneswar.

44. Brajendra High School,
Nayagarh.

45. Peoples High School,
Chanahatta, Khurda

46. Govt. High School(Girls),
Banki.

DEVELOPMENT AND TRYOUT OF A TRAINING PACKAGE FOR TEACHING MATHEMATICS AT SECONDARY LEVEL HELD FROM 18.11.96 TO 22.11.96.

LIST OF PARTICIPANTS

1. Shyama Sundar Sagadia,
   Asst.Teacher,
   Erabang High School,

2. Sridhar Chinara,
   Headmaster,
   N.S.Police High School,
   Keonjhar,Dt.Keonjhar.

3. Prafulla Kumar Maharana,
   Asst.Teacher,
   Govt. Boys' High School,
   Unit-IX, Bhubaneswar,
   Dist.- Khurda.

4. Braja Kishore Rout,
   Headmaster,
   Govt. High School,Tomka,
   Dist.- Jajpur

5. Ramakanta Samal,
   Headmaster I/c
   Derabis High School,
   At/P.O. Derabis
   Dist:- Kendrapara

6. Pramod Kumar Pradhan,
   Asst.Teacher,
   Panchayat High School,
   At/P.O. Gengutia,
   Dist:- Dhenkanal

7. Sisir Kumar Satapathy,
   Headmaster,
   Agrahat High School,
   At/P.O. Agrahat,
   Dist:- Cuttack

8. Umakanta Nanda,
   Science Teacher,
   Kalinga Vidyapitha,
   Choudwar

9. Pradipta Kishor Beura,
   Science Teacher,
   Govt. High School,
   Bisinahakani,
   P.O. Garudagan,
   Via:- Kotasahi,
   Dist:- Cuttack-754022

External Resource Persons

1. Rama Chandra Swain,
   Asst.Teacher (Retd),
   S.B.High School,Cuttack

2. Dr.Bijoy Kumar Khuntia
   Ranihat High School,
   Cuttack - 753 001

3. Sreekanta Ghose,
   At/P.O. Rairangpur,
   Dist:- Mayurbhanj
   Pin - 757 043

4. Brundaban Singh,
   Retd. Headmaster,
   New Friends' Colony,
   Cuttack - 753001

5. Nagendra Kumar Mishra,
   Asst.Teacher,
   P.M.Academy, Cuttack.

6. Nalini Kanta Mishra,
   Mathematics Expert,
   Board of Secondary Education,
   Orissa, Cuttack - 1

7. Madan Mohan Mohanty

8. Dr.S.Padhy,
   Dept. of Mathematics,
   Utkal University,Bhubaneswar.

9. Dr.L.N.Sahoo,
   Dept. of Statistics,
   Utkal University,Bhubaneswar

10. Prof.G.Das,
    Dept. of Mathematics,
    Utkal University,Bhubaneswar

Internal Resource Persons

1. Dr.D.K.Bhattacharjee,
   Principal,RIE,Bhubaneswar.

2. Dr.S.Dutta,
   Dean,RIE, Bhubaneswar.

3. Dr.D.C.Sahoo,
   Reader in Mathematics,
   RIE,Bhubaneswar.

4. Dr.P.S.Tripathi,
   Reader & Prog.Coordinator,
   RIE,Bhubaneswar.

5. Dr.A.D.Tewari,
   Sr.Lecturer in Education,
   RIE, Bhubaneswar.

APPENDIX-III

DEVELOPMENT AND TRYOUT OF A TRAINING PACKAGE FOR TEACHING MATHEMATICS
AT SECONDARY LEVEL

TIME TABLE

Venue:- RIE, Bhubaneswar

Duration.-November 18 to 22,1996

Date/Day	Forenoon Session 10AM - 11AM	11AM - 12 Noon	12 Noon - 1 PM	1PM - 2PM	Afternoon Session 2PM - 3PM	3PM - 4PM	4PM - 5 PM
18.11.96 Monday	Registration	Inauguration	Aims & Objectives of the Workshop	LUNCH	Discussion on the identified HARD SPOTS in Mathematics.		Group formation for development of material.
19.11.96 Tuesday	............Group Work			-do-	............Group Work		
20.11.96 Wednesday	General Session - Presentation and discussion on Training material developed in groups.			-do-	............Group Work		
21.11.96 Thursday	.... Group Work....................			-do-	General Session - Presentation and Discussion on training material developed in groups.		
22.11.96 Friday	.... Finalisation of Group reports.....			-do-	Evaluation Valedictory		Disbursement of TA/DA.

P. Tripathi

(P.S. Tripathi)
Programme Co-ordinator

Appendix - IV

REVIEW AND FINALISATION OF TRAINING PACKAGE FOR TEACHING MATHEMATICS AT SECONDARY LEVEL HELD FROM 10.3.97 TO 14.3.97.

LIST OF RESOURCE PERSONS

1. Rama Chandra Hota,
   Sadashiva Nivas Gopalmal
   P.O. Budharaja, Sambalpur-4

2. Brundaban Singh,
   Ex-Headmaster,
   Secondary Board High School
   New Friends' Colony,
   Cuttack-1

3. Rama Chandra Swain,
   Retd. Asst.Teacher,
   Secondary Board High School,
   Cuttack-1

4. Subhas Chandra Misra,
   Headmaster,
   R.N.Bidyapitha, Khuntuni,
   Cuttack.

5. Nagendra Kumar Mishra,
   Teacher,
   P.M.Academy, Cuttack.

6. Nalini Kanta Mishra
   Jr.Expert in Teaching Math.
   B.S.E., Orissa, Cuttack.

7. Dr.Bijoy Kumar Khuntia,
   Tinigharia
   Cuttack - 753 004

8. Sreekanta Ghose,
   At/P.O. Rairangpur,
   Mayurbhanj, Orissa.

LIST OF EXPERTS (EXTERNAL/INTERNAL)

1. Dr.G.Das,
   Prof.of Mathematics,
   Utkal University,
   Vani Vihar, Bhubaneswar.

2. S.Padhy,
   Reader, Dept. of Math.,
   Utkal University,
   Vani Vihar, Bhubaneswar.

3. M.M. Mohanty,
   Deputy Secretary,
   Board of Secondary
   Education, Orissa,
   Cuttack.

4. Dr.I.S.Tripathi,
   Reader in Mathematics,
   and Prog.Coordinator
   RIE, Bhubaneswar.

5. Dr.A.D.Tewari,
   Sr.Lecturer in Education,
   RIE, Bhubaneswar.

6. Dr.D.C. Sahoo,
   Reader in Mathematics,
   RIE, Bhubaneswar.

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Appendix - V

REVIEW AND FINALISATION OF TRAINING PACKAGE FOR TEACHING MATHEMATICS AT SECONDARY LEVEL

TIME TABLE

Duration:- March 10 to 14,1997 — Venue:- RIE,Bhubaneswar

Date/Day	10AM - 11 AM (Forenoon session)	11 AM - 12 Noon	12Noon - 1PM	1PM - 2PM	2PM - 3 PM (Afternoon session)	3 PM - 4 PM	4PM - 5 PM
10.3.97 Monday	Registration	Inauguration	Aims & Objectives of the workshop.	L	......Review and finalisation of the Trg. materials on Statistics		
11.3.97 Tuesday	.....Review of the training material on Geometry.			U N	.....Finalisation of the training material on Geometry		
12.3.97 Wednesday	.....Review of the training Material in Algebra			C H	.....Finalisation of the training material on Algebra........		
13.3.97 Thursday	......Review of the training Material on Computing			B	.....Finalisation of the training Material on Computing.......		
14.3.97 Friday	....Finalis tion of the training materials....			R E A K	Evaluation	Valedictory	Disbursement of TA/DA

(P.S. Tripathi)
Programme Co-Crdinator
RIE,Bhubaneswar